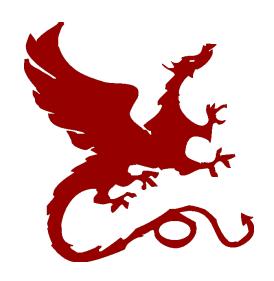
Algorithms for NLP



Classification III

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Slides: Dan Klein – UC Berkeley

The Perceptron, Again

- Start with zero weights
- Visit training instances one by one
 - Try to classify

$$\hat{\mathbf{y}} = \underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{arg max}} \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y})$$

- If correct, no change!
- If wrong: adjust weights

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}_i(\mathbf{y}_i^*)$$
 $\mathbf{w} \leftarrow \mathbf{w} - \mathbf{f}_i(\hat{\mathbf{y}})$
 $\mathbf{w} \leftarrow \mathbf{w} + (\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\hat{\mathbf{y}}))$
 $\mathbf{w} \leftarrow \mathbf{w} + \Delta_i(\hat{\mathbf{y}})$ mistake vectors

Perceptron Weights

What is the final value of w?

 $\mathbf{w} \leftarrow \mathbf{w} + \Delta_i(\mathbf{y})$

- Can it be an arbitrary real vector?
- No! It's built by adding up feature vectors (mistake vectors).

$$\mathbf{w} = \Delta_i(\mathbf{y}) + \Delta_{i'}(\mathbf{y}') + \cdots$$

$$\mathbf{w} = \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y}) \Delta_i(\mathbf{y})$$
 mistake counts

 Can reconstruct weight vectors (the primal representation) from update counts (the dual representation) for each i

$$\alpha_i = \langle \alpha_i(\mathbf{y}_1) \ \alpha_i(\mathbf{y}_2) \ \dots \ \alpha_i(\mathbf{y}_n) \rangle$$

Dual Perceptron

Track mistake counts rather than weights

$$\mathbf{w} = \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \Delta_i(\mathbf{y})$$

- Start with zero counts (α)
- For each instance x
 - Try to classify

$$\widehat{\mathbf{y}} = \underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\text{arg max}} \mathbf{w}^{\top} \mathbf{f}(\mathbf{y})$$

$$\hat{\mathbf{y}} = \underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x}_i)}{\arg \max} \sum_{i',\mathbf{y}'} \alpha_{i'}(\mathbf{y}') \Delta_{i'}(\mathbf{y}')^{\top} \mathbf{f}_i(\mathbf{y})$$

- If correct, no change!
- If wrong: raise the mistake count for this example and prediction

$$\alpha_i(\hat{\mathbf{y}}) \leftarrow \alpha_i(\hat{\mathbf{y}}) + 1$$

$$\mathbf{w} \leftarrow \mathbf{w} + \Delta_i(\hat{\mathbf{y}})$$

Dual / Kernelized Perceptron

How to classify an example x?

$$score(\mathbf{y}) = \mathbf{w}^{\top} \mathbf{f}_{i}(\mathbf{y}) = \left(\sum_{i',\mathbf{y}'} \alpha_{i'}(\mathbf{y}') \Delta_{i'}(\mathbf{y}')\right)^{\top} \mathbf{f}_{i}(\mathbf{y})$$

$$= \sum_{i',\mathbf{y}'} \alpha_{i'}(\mathbf{y}') \left(\Delta_{i'}(\mathbf{y}')^{\top} \mathbf{f}_{i}(\mathbf{y})\right)$$

$$= \sum_{i',\mathbf{y}'} \alpha_{i'}(\mathbf{y}') \left(\mathbf{f}_{i'}(\mathbf{y}_{i'}^{*})^{\top} \mathbf{f}_{i}(\mathbf{y}) - \mathbf{f}_{i'}(\mathbf{y}')^{\top} \mathbf{f}_{i}(\mathbf{y})\right)$$

$$= \sum_{i',\mathbf{y}'} \alpha_{i'}(\mathbf{y}') \left(K(\mathbf{y}_{i'}^{*},\mathbf{y}) - K(\mathbf{y}',\mathbf{y})\right)$$

 If someone tells us the value of K for each pair of candidates, never need to build the weight vectors

Issues with Dual Perceptron

 Problem: to score each candidate, we may have to compare to all training candidates

$$score(\mathbf{y}) = \sum_{i',\mathbf{y}'} \alpha_{i'}(\mathbf{y}') \left(K(\mathbf{y}_{i'}^*, \mathbf{y}) - K(\mathbf{y}', \mathbf{y}) \right)$$

- Very, very slow compared to primal dot product!
- One bright spot: for perceptron, only need to consider candidates we made mistakes on during training
- Slightly better for SVMs where the alphas are (in theory) sparse
- This problem is serious: fully dual methods (including kernel methods) tend to be extraordinarily slow
- Of course, we can (so far) also accumulate our weights as we go...



Kernels: Who Cares?

- So far: a very strange way of doing a very simple calculation
- "Kernel trick": we can substitute any* similarity function in place of the dot product
- Lets us learn new kinds of hypotheses

* Fine print: if your kernel doesn't satisfy certain technical requirements, lots of proofs break.

E.g. convergence, mistake bounds. In practice, illegal kernels *sometimes* work (but not always).

Some Kernels

- Kernels implicitly map original vectors to higher dimensional spaces, take the dot product there, and hand the result back
- Linear kernel:

$$K(x, x') = x' \cdot x' = \sum_{i} x_i x_i'$$

• Quadratic kernel:

$$K(x, x') = (x \cdot x' + 1)^{2}$$
$$= \sum_{i,j} x_{i}x_{j} x'_{i}x'_{j} + 2\sum_{i} x_{i} x'_{i} + 1$$

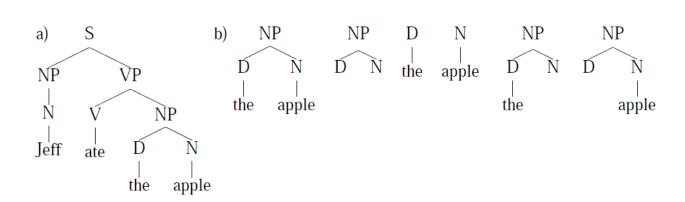
RBF: infinite dimensional representation

$$K(x, x') = \exp(-||x - x'||^2)$$

Discrete kernels: e.g. string kernels, tree kernels

Tree Kernels

[Collins and Duffy 01]



- Want to compute number of common subtrees between T, T'
- Add up counts of all pairs of nodes n, n'
 - Base: if n, n' have different root productions, or are depth 0:

$$C(n_1, n_2) = 0$$

Base: if n, n' are share the same root production:

$$C(n_1, n_2) = \lambda \prod_{j=1}^{nc(n_1)} (1 + C(ch(n_1, j), ch(n_2, j)))$$

Kernelized SVM (trust me)

Primal formulation:

$$\min_{\mathbf{w}, \xi} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i} \xi_i$$
$$\forall i, \mathbf{y} \quad \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \ge \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i$$

$$\mathbf{w} = \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y}) \right)$$

Dual formulation:

$$\min_{\alpha \ge 0} \quad \frac{1}{2} \left\| \sum_{i,y} \alpha_i(y) \left(f_i(y_i^*) - f_i(y) \right) \right\|_2^2 - \sum_{i,y} \alpha_i(y) \ell_i(y)$$

$$\forall i \ \sum_{y} \alpha_i(y) = C$$

Dual Formulation for SVMs

We want to optimize: (separable case for now)

$$\begin{aligned} & \min_{\mathbf{w}} & \frac{1}{2} ||\mathbf{w}||^2 \\ & \forall i, \mathbf{y} & \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) \end{aligned}$$

- This is hard because of the constraints
- Solution: method of Lagrange multipliers
- The Lagrangian representation of this problem is:

$$\min_{\mathbf{w}} \max_{\alpha \geq 0} \quad \Lambda(\mathbf{w}, \alpha) = \frac{1}{2} ||\mathbf{w}||^2 - \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) - \ell_i(\mathbf{y}) \right)$$

 All we've done is express the constraints as an adversary which leaves our objective alone if we obey the constraints but ruins our objective if we violate any of them

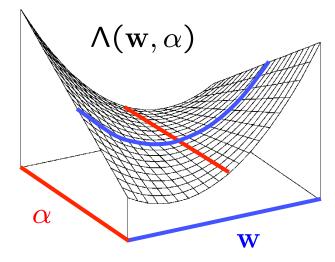
Lagrange Duality

We start out with a constrained optimization problem:

$$f(\mathbf{w}^*) = \min_{\mathbf{w}} f(\mathbf{w})$$
$$g(\mathbf{w}) \ge 0$$

We form the Lagrangian:

$$\Lambda(\mathbf{w}, \boldsymbol{\alpha}) = f(\mathbf{w}) - \boldsymbol{\alpha} g(\mathbf{w})$$



• This is useful because the constrained solution is a saddle point of Λ (this is a general property):

$$f(\mathbf{w}^*) = \min_{\mathbf{w}} \max_{\alpha \ge 0} \Lambda(\mathbf{w}, \alpha) = \max_{\alpha \ge 0} \min_{\mathbf{w}} \Lambda(\mathbf{w}, \alpha)$$
Primal problem in w

Dual problem in α

Dual Formulation II

Duality tells us that

$$\min_{\mathbf{w}} \max_{\alpha \geq 0} \frac{1}{2} ||\mathbf{w}||^2 - \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) - \ell_i(\mathbf{y}) \right)$$

has the same value as

max
$$\min_{\alpha \geq 0} \frac{1}{\mathbf{v}} \frac{1}{2} ||\mathbf{w}||^2 - \sum_{i,\mathbf{v}} \alpha_i(\mathbf{y}) \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) - \ell_i(\mathbf{y}) \right)$$

- This is useful because if we think of the α 's as constants, we have an unconstrained min in w that we can solve analytically.
- Then we end up with an optimization over α instead of w (easier).

Dual Formulation III

• Minimize the Lagrangian for fixed α 's:

$$\Lambda(\mathbf{w}, \alpha) = \frac{1}{2} ||\mathbf{w}||^2 - \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) - \ell_i(\mathbf{y}) \right)
\frac{\partial \Lambda(\mathbf{w}, \alpha)}{\partial \mathbf{w}} = \mathbf{w} - \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y}) \right)
\frac{\partial \Lambda(\mathbf{w}, \alpha)}{\partial \mathbf{w}} = 0 \qquad \qquad \mathbf{w} = \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y}) \right)$$

• So we have the Lagrangian as a function of only α 's:

$$\min_{\alpha \ge 0} Z(\alpha) = \frac{1}{2} \left\| \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y}) \right) \right\|^2 - \sum_{i, \mathbf{y}} \alpha_i(\mathbf{y}) \ell_i(\mathbf{y})$$

Primal vs Dual SVM

Primal formulation:

$$\min_{\mathbf{w}, \xi} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i} \xi_i$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \ge \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i$$

$$\mathbf{w} = \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y}) \right)$$

Dual formulation:

$$\min_{\alpha \ge 0} \quad \frac{1}{2} \left\| \sum_{i,y} \alpha_i(y) \left(f_i(y_i^*) - f_i(y) \right) \right\|_2^2 - \sum_{i,y} \alpha_i(y) \ell_i(y)$$

$$\forall i \ \sum_{y} \alpha_i(y) = C$$

Learning SVMs (Primal)

Primal formulation:

$$\min_{\mathbf{w}, \xi} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i} \xi_i$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \ge \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i$$



$$\min_{w} \frac{1}{2} ||w||_{2}^{2} + C \sum_{i} \left(\max_{y} \left(w^{\top} f_{i}(y) + \ell_{i}(y) \right) - w^{\top} f_{i}(y_{i}^{*}) \right)$$

Learning SVMs (Primal)

Primal formulation:

$$\min_{w} \frac{1}{2} \|w\|_{2}^{2} + C \sum_{i} \left(\max_{y} \left(w^{\top} f_{i}(y) + \ell_{i}(y) \right) - w^{\top} f_{i}(y_{i}^{*}) \right)$$

Loss-augmented decode: $\bar{y} = \operatorname{argmax}_y \left(w^\top f_i(y) + \ell_i(y) \right)$

$$\min_{w} \frac{1}{2} \|w\|_{2}^{2} + C \sum_{i} \left(w^{\top} f_{i}(\bar{y}) + \ell_{i}(\bar{y}) - w^{\top} f_{i}(y_{i}^{*}) \right)$$

$$\nabla_w = w + C \sum_i \left(f_i(\bar{y}) - f_i(y_i^*) \right)$$

Use general subgradient descent methods! (Adagrad)

Learning SVMs (Dual)

• We want to find α which minimize

$$\min_{\alpha \ge 0} \frac{1}{2} \left\| \sum_{i,y} \alpha_i(y) \left(f_i(y_i^*) - f_i(y) \right) \right\|_2^2 - \sum_{i,y} \alpha_i(y) \ell_i(y)$$

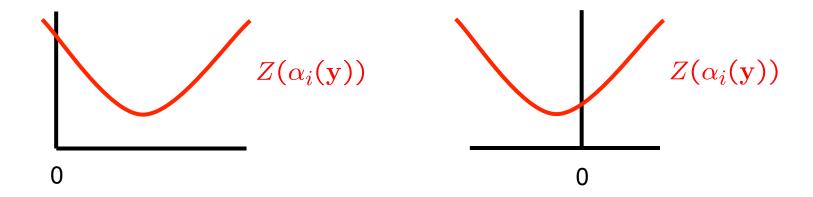
$$\forall i \sum_{y} \alpha_i(y) = C$$

- This is a quadratic program:
 - Can be solved with general QP or convex optimizers
 - But they don't scale well to large problems
 - Cf. maxent models work fine with general optimizers (e.g. CG, L-BFGS)
- How would a special purpose optimizer work?

Coordinate Descent I (Dual)

$$\min_{\alpha \ge 0} \quad \frac{1}{2} \left\| \sum_{i,y} \alpha_i(y) \left(f_i(y_i^*) - f_i(y) \right) \right\|_2^2 - \sum_{i,y} \alpha_i(y) \ell_i(y)$$

- Despite all the mess, Z is just a quadratic in each $\alpha_i(y)$
- Coordinate descent: optimize one variable at a time



If the unconstrained argmin on a coordinate is negative, just clip to zero...

Coordinate Descent II (Dual)

 Ordinarily, treating coordinates independently is a bad idea, but here the update is very fast and simple

$$\alpha_i(\mathbf{y}) \leftarrow \max \left(0, \alpha_i(\mathbf{y}) + \frac{\ell_i(\mathbf{y}) - \mathbf{w}^\top \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y})\right)}{\left\|\left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y})\right)\right\|^2}\right)$$

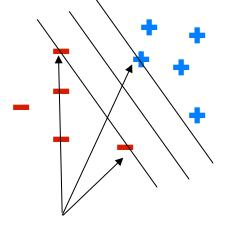
- So we visit each axis many times, but each visit is quick
- This approach works fine for the separable case
- For the non-separable case, we just gain a simplex constraint and so we need slightly more complex methods (SMO, exponentiated gradient)

$$\forall i, \quad \sum_{\mathbf{y}} \alpha_i(\mathbf{y}) = C$$

What are the Alphas?

Each candidate corresponds to a primal constraint

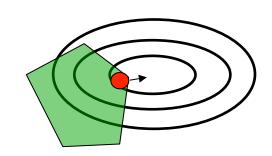
$$\min_{\mathbf{w}, \xi} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i} \xi_i$$
$$\forall i, \mathbf{y} \quad \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \ge \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i$$



Support vectors

- In the solution, an $\alpha_i(y)$ will be:
 - Zero if that constraint is inactive
 - Positive if that constrain is active
 - i.e. positive on the support vectors
- Support vectors contribute to weights:

$$\mathbf{w} = \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y}) \left(\mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{f}_i(\mathbf{y}) \right)$$



Structure



Handwriting recognition

X

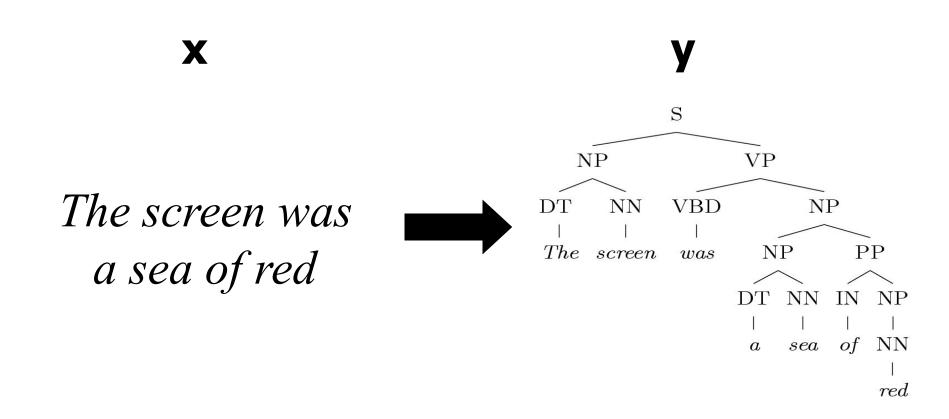


Sequential structure

[Slides: Taskar and Klein 05]



CFG Parsing



Recursive structure

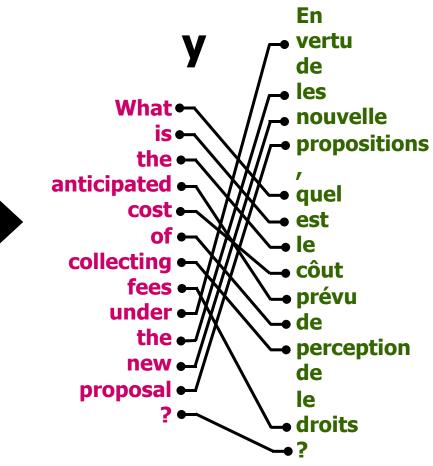


Bilingual Word Alignment

X

What is the anticipated cost of collecting fees under the new proposal?

En vertu de nouvelle propositions, quel est le côut prévu de perception de les droits?



Combinatorial structure

Structured Models

$$prediction(\mathbf{x}, \mathbf{w}) = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} score(\mathbf{y}, \mathbf{w})$$



Assumption:

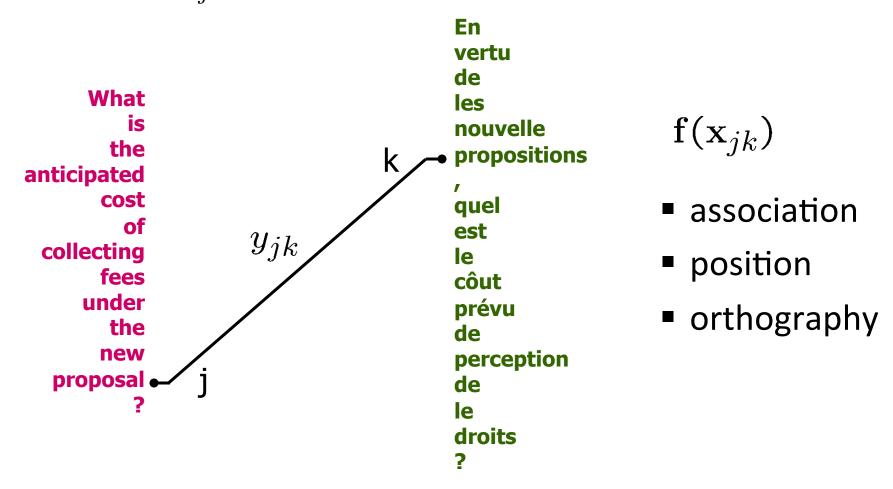
$$score(\mathbf{y}, \mathbf{w}) = \mathbf{w}^{\top} \mathbf{f}(\mathbf{y}) = \sum_{p} \mathbf{w}^{\top} \mathbf{f}(\mathbf{y}_{p})$$

Score is a sum of local "part" scores

Parts = nodes, edges, productions

Bilingual word alignment

$$\sum_{y_{jk} \in \mathbf{y}} \mathbf{w}^{\top} \mathbf{f}(\mathbf{x}_{jk}) = \mathbf{w}^{\top} \mathbf{f}(\mathbf{x}, \mathbf{y})$$



Efficient Decoding

Common case: you have a black box which computes

$$prediction(\mathbf{x}) = \underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{arg \, max \, \mathbf{w}^{\top} \mathbf{f}(\mathbf{y})}$$

at least approximately, and you want to learn w

- Easiest option is the structured perceptron [Collins 01]
 - Structure enters here in that the search for the best y is typically a combinatorial algorithm (dynamic programming, matchings, ILPs, A*...)
 - Prediction is structured, learning update is not

Structured Margin (Primal)

Remember our primal margin objective?

$$\min_{w} \frac{1}{2} ||w||_{2}^{2} + C \sum_{i} \left(\max_{y} \left(w^{\top} f_{i}(y) + \ell_{i}(y) \right) - w^{\top} f_{i}(y_{i}^{*}) \right)$$

Still applies with structured output space!

Structured Margin (Primal)

Just need efficient loss-augmented decode:

$$\bar{y} = \operatorname{argmax}_y \left(w^{\top} f_i(y) + \ell_i(y) \right)$$

$$\min_{w} \frac{1}{2} \|w\|_{2}^{2} + C \sum_{i} \left(w^{\top} f_{i}(\bar{y}) + \ell_{i}(\bar{y}) - w^{\top} f_{i}(y_{i}^{*}) \right)$$

$$\nabla_w = w + C \sum_i \left(f_i(\bar{y}) - f_i(y_i^*) \right)$$

Still use general subgradient descent methods! (Adagrad)

Structured Margin (Dual)

Remember the constrained version of primal:

$$\min_{\mathbf{w}, \xi} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i} \xi_i$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \ge \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i$$

Dual has a variable for every constraint here

Full Margin: OCR

We want:

$$\operatorname{arg\,max}_{\mathbf{y}} \ \mathbf{w}^{\top} \mathbf{f}(\mathbf{y}, \mathbf{y}) = \operatorname{``brace''}$$

Equivalently:

$$\begin{array}{lll} w^\top f(\text{brace}^{,}\text{``brace''}) &> & w^\top f(\text{brace}^{,}\text{``aaaaa''}) \\ w^\top f(\text{brace}^{,}\text{``brace''}) &> & w^\top f(\text{brace}^{,}\text{``aaaab''}) \\ & & \cdots \\ w^\top f(\text{brace}^{,}\text{``brace''}) &> & w^\top f(\text{brace}^{,}\text{``zzzzz''}) \end{array} \right\} \text{a lot!}$$

Parsing example

We want:

arg max
$$_{y}$$
 $w^{ op}f($ 'It was red' $,y)$ $=$ $A^{\S}_{c}B_{D}$

Equivalently:

Alignment example

We want:

Equivalently:

$$\begin{array}{c} w^\top f(\begin{subarray}{c} \begin{subarray}{c} \begin{subar$$

Cutting Plane (Dual)

- A constraint induction method [Joachims et al 09]
 - Exploits that the number of constraints you actually need per instance is typically very small
 - Requires (loss-augmented) primal-decode only
- Repeat:
 - Find the most violated constraint for an instance:

$$\forall \mathbf{y} \quad \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

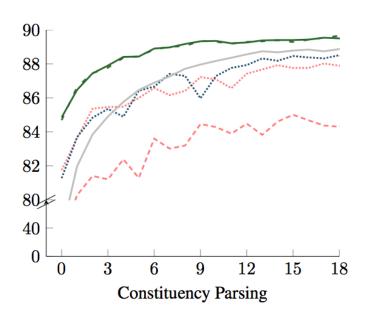
$$\arg\max_{\mathbf{y}} \mathbf{w}^{\top} \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

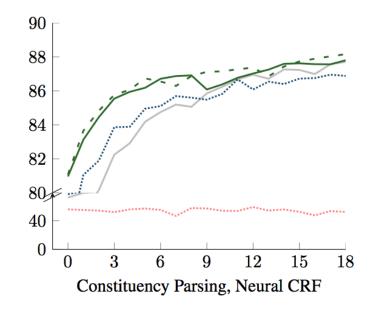
 Add this constraint and resolve the (non-structured) QP (e.g. with SMO or other QP solver)



Comparison

Oct 20	Structured Classification III	
Oct 25	Structured Classification IV	J+M 16, 18, 19, Adagrad, Subgradient SVM





Option 0: Reranking

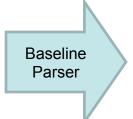
[e.g. Charniak and Johnson 05]

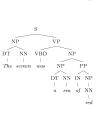
Input

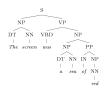
N-Best List (e.g. n=100)

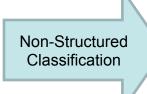
Output

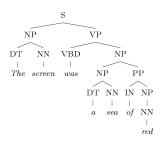
 χ = "The screen was a sea of red."







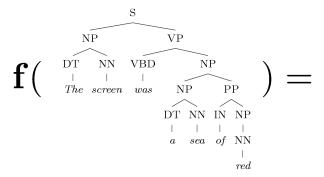




Reranking

Advantages:

- Directly reduce to non-structured case
- No locality restriction on features



Disadvantages:

- Stuck with errors of baseline parser
- Baseline system must produce n-best lists
- But, feedback is possible [McCloskey, Charniak, Johnson 2006]