

# Cut Elimination and Second Order Quantifier Elimination (Abstract of Tutorial)

Alessandra Palmigiano

with Giuseppe Greco, Minghui Ma, Apostolos Tzimoulis, Zhiguang Zhao

Delft University of Technology, Netherlands

Display calculi, pioneered by Belnap [1], are a proof-theoretic framework generalizing Gentzen's sequent calculi, which have succeeded in endowing a large class of modal logics with cut-free sequent calculi in a uniform and modular way. The robustness and modularity of display calculi are rooted in a general methodology for proving cut-elimination, which identifies conditions on the design of sequent calculi which guarantee the success of a certain uniform strategy for syntactic cut elimination.

Recently, systematic connections have been established between algorithmic correspondence theory, well known from the area of modal logic, and the theory of display calculi. These connections originate from some seminal observations made by Kracht [5], in the context of his characterization of the modal axioms which can be effectively transformed into 'analytic' structural rules of display calculi. In this context, a rule is 'analytic' if adding it to a display calculus preserves Belnap's cut-elimination theorem.

The present tutorial illustrates these connections. Specifically, after introducing (proper) display calculi and discussing the uniform strategy for their cut elimination, I will discuss how the two main tools of unified correspondence theory [3], [2] (namely, (a) the *ALBA algorithm* for second order quantifier elimination, and (b) the syntactically defined class of *inductive inequalities* in each logical/algebraic signature of normal distributive lattice expansions) can be used to produce analytic calculi for a certain subclass of inductive formulas (the *analytic inductive inequalities*), and to exhaustively characterize this subclass as the class of the 'properly displayable' logics.

Time permitting, I will also discuss how the methodology of *multi-type calculi* [4] can be used to circumvent this exhaustive characterization, and export these techniques also to *non analytic* logics.

## References

1. N. Belnap. Display logic. *Journal of Philosophical Logic*, 11:375–417, 1982.
2. W. Conradie, S. Ghilardi, and A. Palmigiano. Unified correspondence. In A. Baltag and S. Smets, editors, *Johan van Benthem on Logic and Information Dynamics*, volume 5 of *Outstanding Contributions to Logic*, pages 933–975. Springer International Publishing, 2014.

*Copyright © 2017 by the paper's authors*

In: P. Koopmann, S. Rudolph, R. Schmidt, C. Wernhard (eds.): *SOQE 2017 – Proceedings of the Workshop on Second-Order Quantifier Elimination and Related Topics, Dresden, Germany, December 6–8, 2017*, published at <http://ceur-ws.org>.

3. W. Conradie and A. Palmigiano. Algorithmic correspondence and canonicity for non-distributive logics. Submitted. ArXiv preprint 1603.08515.
4. S. Frittella, G. Greco, A. Kurz, A. Palmigiano, and V. Sikimić. Multi-type sequent calculi. In M. Z. A. Indrzejczak and J. Kaczmarek, editors, *Proceedings of Trends in Logic XIII*, pages 81–93. Lodz University Press, 2014.
5. M. Kracht. Power and weakness of the modal display calculus. In *Proof theory of modal logic*, volume 2 of *Applied Logic Series*, pages 93–121. Kluwer, 1996.