Trend Detection in Short Time Series Using Discrete Wavelet Transform

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Abstract

The paper proposes a method for detecting the trend component in short time series using the wavelet test. The test for trend is based on the wavelet decomposition of a time series using the Haar wavelet. Numerical studies have shown that the proposed method allows detecting the presence of various trend components in short time series (from 8 values). The results show the advantages of the wavelet test over many known statistical ones when detecting a trend in time series of small length. The sample value of the test can be used as a feature for the classification or clustering of time series by machine learning methods.

Keywords [1]

Time series, trend, discrete wavelet transform, Haar wavelet, trend test statistic

1. Introduction

During the study of time series, the question of the regularities of their dynamics over a long period of time is of great importance. Cognition of regularities of changes in time is a complicated and time-consuming research procedure since any phenomenon under study is formed by many factors acting in different directions. One of the most important tasks in the study and analysis of time series is to identify and statistically evaluate the main trend of the process and deviations from it.

One of the complex concepts of time series analysis is the concept of trend. However, it should be noted that the trend of a time series is a rather conventional concept. A trend is understood as a regular, non-random component of a time series (usually monotonic), which can be calculated according to a well-defined explicit rule. The trend of a real time series is often related to the action of natural (e.g., physical) acts or some other objective regularities. However, it is quite difficult to uniquely divide a time series into regular parts (trend) and fluctuations (residual) [1,2]. Therefore, in practice, in various fields of science and technology, in particular related to infocommunications and information technology, it is usually assumed that a trend is some function or curve of a fairly simple type (linear, quadratic, etc.) that describes the "average behavior" of a series or process [3-6].

An effective tool for studying time series is the multiresolution wavelet analysis, which allows decomposing a time series on an orthogonal basis, formed by shifts and multiresolution copies of a wavelet function [7]. The analyzed time series is divided into two components: approximation and detail, with their subsequent splitting to change the decomposition level of the series.

Recently, a series of works have been published where the trend component was determined using discrete wavelet transform (DWT). In [8], a trend analysis and variance estimation at different frequencies in precipitation series based on wavelet analysis were carried out, the results of which were used to cluster groups of precipitation and meteorological systems. The authors of [9] developed a

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method for modeling time series with a variable structure based on wavelets for locally stationary processes with the inclusion of trend components. The authors also proposed a method for processing the limit of a time series, which applies to data of any length that has trends, by calculating a discrete wavelet transform. The authors of the study [10] used wavelet analysis to determine the time trend of air temperature. The influence of the trend on the flow of forest streams around their average values was also analyzed. In [11], the authors conduct research to identify trends based on discrete wavelet transform. The authors propose a test for determining the best wavelet type and decomposition level that provides the best wavelet approximation to the original time series.

The authors of [12] studied wavelet transform methods for the analysis of non-stationary time series and focused on the extraction of second-order components from non-stationary time series and their application in various applications. In [13], the authors proposed a methodology for trend analysis of non-stationary time series based on wavelet analysis, taking into account the best characteristics of wavelet transform and their impact on trend detection. The authors considered various types of trends and discrete wavelets.

To identify trends in time series, the authors of [14] developed a method based on discrete wavelet transform and k-means clustering. Based on the wavelet reconstruction of the signal, it is possible to determine the most significant interval of change in the dynamics of the time series. The authors of [15] analyzed the structure of time series of air temperature, precipitation, and river runoff using wavelet analysis. This made it possible to investigate the nature of river flow patterns and identify dependencies on natural and artificial processes.

It should be noted that most of the research works use the discrete wavelet transform to identify the trend component on a long time interval when the time series has a sufficiently big length. However, many tasks require determining the presence of a trend in a very short series, starting from a tenth value. For example, such problems include the detection of gamma-ray bursts [16-18]. The duration of a typical gamma-ray burst is a few seconds, during this time period it is possible to obtain a maximum of 60 values, the sequence of which has a trend. Thus, one of the actual tasks of time series analysis is the detection of a trend component in short time series [19,20].

The objective of this study is to develop a test to detect the presence of a trend in short time series based on the discrete wavelet transform.

2. Problem statement

The task of this study is to develop a test to detect the presence of a trend in short time series, which is based on the decomposition of the time series using the discrete wavelet transform. Consider that the time series consists of two components, trend and white noise:

$$S(t) = T(t) + \xi(t),$$

where $S(t)$ is the input time series, $t = \overline{1, N}$; $T(t)$ is a trend, generally being some deterministic function of time (linear, polynomial, exponential, logarithmic, etc.); t is white noise with normal distribution $N(0,\sigma)$.

It is necessary to develop a statistic test $K(S,N,wavelet)$, where $S(t)$ is the input time series, N is a length of time series, *wavelet* is a type of wavelet function used in discrete wavelet decomposition of time series. The null hypothesis H_0 is assumed that the series of observations does not contain a trend, the opposite hypothesis H_1 is, that the time series contains a trend. The acceptance or rejection of the non-trend hypothesis is carried out with the given significance level α.

3. Discrete wavelet transform

Wavelet analysis is a frequency-space analysis of signals. The idea of using wavelets is to decompose a signal $X(t)$ using a basis formed by shifts and different-scale copies of the basis (mother) prototype function $\psi(t)$. The basic functions $\psi(t)$ are called wavelets if they are defined on the space of complex-valued functions with bounded energy, oscillate around the abscissa axis, rapidly converge to zero, and satisfy the condition $\int_{-\infty}^{\infty} \psi(t)dt = 0$.

There are continuous and discrete wavelet transforms. Continuous wavelet transform is a decomposition of a signal by all possible shifts and compressions/extensions of some function - wavelet:

$$C(a,b) = \int_{-\infty}^{\infty} s(t)\psi(a,b,t)dt, \quad a,b \in R, \quad a \neq 0,$$

where wavelets $\psi(a,b,t)$ are scaled and shifted copies of the generating wavelet $\psi(t)$. The variable 'a' defines the scale of the wavelet and is the inverse of the frequency in the Fourier transforms, and the variable 'b' is the shift of the wavelet along the signal from the starting point in its definition range, whose scale completely repeats the timeline of the analyzed signal.

DWT provides enough information both for signal analysis and for its synthesis, being at the same time economical in the number of operations and in the required memory. The DWT operates with discrete values of parameters a and b, which are set, as a rule, in the form of power functions:

$$a = 2^{-j}, b = k \cdot 2^{-j}, \quad j,k \in Z,$$

where Z is a space of integers, j is a scale parameter, k is a shift parameter.

One of the fundamental ideas of DWT signals is to divide the studied signal into two components - approximation and detail - with their subsequent fragmentation in order to change the level of signal decomposition [7,21]. This is possible both in the temporal and in the frequency range of signal representation [22,23]. The number of practically used wavelets by scale coefficient j defines the level of signal decomposition. Usually during the processing of the time series $X(t)$ of volume n the wavelet analysis is performed by decomposing the series into functions of detail of different scale j $(0 < j \le N)$ with maximal value $N = [\log_2 n]$. The value of the scale index $j = 0$ corresponds to the case of maximum scale - the most accurate approximation, that is equal to the initial series $X(t)$, consisting of $n_0 = 2^N$ counts. With increasing j, there is a transition to a rougher resolution.

Discrete wavelets are used in pairs with the associated discrete scaling functions $\phi_{J,k}(t)$. The decomposition of time series, performed by using DWT, consists in splitting the studied series into two components: approximation and detail components, with the further similar splitting of the approximation component to the specified decomposition level. The time series $X(t)$ is represented as a sum of approximation $\mathrm{approx}_N(t)$ and details $\mathrm{detail}_j(t)$:

$$X(t) = \mathrm{approx}_N(t) + \sum_{j=1}^{N} \mathrm{detail}_j(t) = \sum_{k=1}^{N_a} \mathrm{apr}(N,k)\varphi_{J,k}(t) + \sum_{j=1}^{N}\sum_{k=1}^{N_j} \det(j,k)\psi_{j,k}(t), \quad (1)$$

where N is the selected maximum decomposition level, N_j is the number of detail coefficients at the level of j, N_a is the number of approximation coefficients at the N level.

For the given mother wavelet ψ and the corresponding scaling function φ, the approximation coefficients $\mathrm{apr}(j,k)$ and the detail coefficients $\det(j,k)$ $d_{j,k}$ of the DWT for the process $X(t)$ are defined as follows:

$$\mathrm{apr}(j,k) = \int_{-\infty}^{\infty} X(t)\varphi_{j,k}(t)dt, \quad \det(j,k) = \int_{-\infty}^{\infty} X(t)\psi_{j,k}(t)dt \quad , \quad (2)$$

where $\varphi_{j,k} = 2^{-j/2}\varphi\left(2^{-j}t - k\right); \quad \psi_{j,k} = 2^{-j/2}\psi\left(2^{-j}t - k\right)$.

The classical form of multiresolution analysis transforms the time series into a hierarchical structure by using wavelet transforms. The hierarchical representation greatly simplifies the analysis of the investigated process. One of the most popular wavelets, also because of its simplicity, is the Haar wavelet. Its mother function ψ and the corresponding scaling function have the form:

$$\varphi(\mathrm{t}) = \begin{cases} 1, 0 \le t < 1; \\ 0, t \notin [0,1). \end{cases}, \qquad \psi(\mathrm{t}) = \begin{cases} 1,\ for\ 0 \le t \le \frac{1}{2}; \\ -1,\ for\ \frac{1}{2} \le t \le 1; \\ 0,\ for\ t < 0, t > 1. \end{cases} \quad (3)$$

and shown in figure 1.

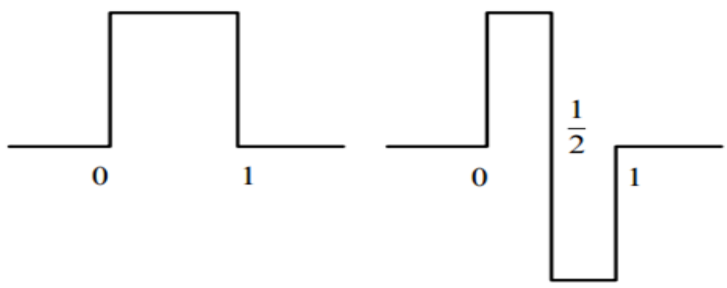

Figure 1: Haar wavelet: (a) mother function ψ , (b) scaling function φ

In this case, the decomposition of the input time series (1) is performed as follows. The input of the realization is the time realization $S(t) = \{S_j\},\ j = \overline{1,n}$. For each pair of elements of the series with the indices *2j* and *2j*+1 let's assign two values:

$$v_j = \frac{s_{2j} + s_{2j+1}}{2}, w_j = \frac{s_{2j} - s_{2j+1}}{2}.$$

These values form approximation $v = \{v_j\}$ and detail components $w = \{w_j\}$ of the original time series $\{S_j\}$. The component $\{v_j\}$ is a roughened version of $\{S_j\}$, and the component $\{w_j\}$ contains the detailed information needed to reconstruct the original series:

$$s_{2j} = v_j + w_j, s_{2j+1} = v_j - w_j, j \in \mathbf{Z}.$$

Signal reconstruction is performed according to the formulas:

$$v_{2j}^{(i+1)} = v_j^{(i)} + w_j^{(i)}, v_{2j+1}^{(i+1)} = v_j^{(i)} - w_j^{(i)}, j \in \mathbf{Z}, i = i_0, i_0 + 1, ..., i_1 - 1.$$

These formulas define the forward and inverse Haar transform of a one-dimensional discrete signal.

Apply a similar operation to the approximation component $\{v_j\}$ and obtain two new approximation and detail components. Then choose the maximum resolution level i_1 . Then recursive formulas for computing the approximation and detail components on the level $i_0 < i_1$ will be as follows

$$v_j^{(i_1)} = s_j, j \in \mathbf{Z}. \quad (4)$$

4. The basic trend components of time series

A trend is a general systematic component that changes consistently over time. The basic mathematical trend models are presented in Table 1:

Table 1
Trends models

Name	Formula
Linear	$y_t = a + b \cdot t$
Exponential	$y_t = a \cdot k^t$
Hyperbolic	$y_t = a + b / t$
Power	$y_t = a \cdot t^b$
Polynomial	$y_t = a + b_1 \cdot t + b_2 \cdot t^2 + \ldots + b_m \cdot t^m$
Logarithmic	$y_t = a + b \cdot \log t$
Logical	$y_t = \dfrac{1}{e^{a+b\cdot t}+1}; y_t = \dfrac{y_{\max} - y_{\min}}{e^{a+b\cdot t}-1} + y_{\min}$

The linear type of trend (Fig. 2) is appropriate for displaying the trend of approximately uniform change in the amplitude of the time series. The reason for such behavior lies in the influence of differently directed and differently accelerated forces of factors, which are mutually averaged, partially mutually extinguished, and the resultant becomes a character close to a uniform one. A polynomial trend (Fig. 3) usually describes data changing smoothly in different directions. A parabolic trend is the most common. In this case, the dynamics of time series is characterized by approximately constant acceleration of absolute changes in the amplitude.

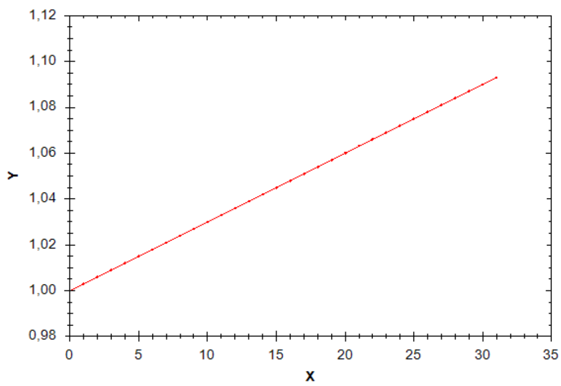

Figure 2: Linear trend

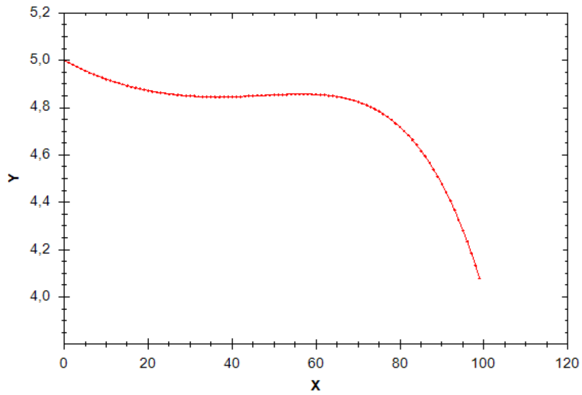

Figure 3: Polynomial trend

Power trends (Fig. 4) are used when the data consists of the results of observations, the values of which increase smoothly with increasing speed. The hyperbolic form of the trend (Fig. 5) is suitable for displaying the trend, and processes limited by the level limit value.

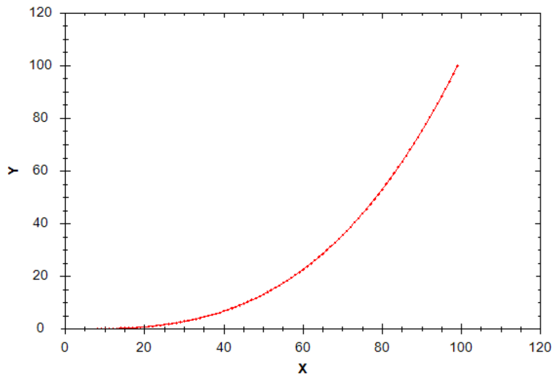

Figure 4: Power trend

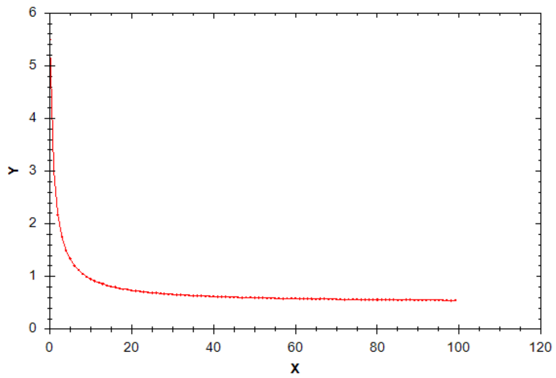

Figure 5: Hyperbolic trend

The logarithmic trend equation (Fig. 6) is used when the process under study leads to deceleration of the index growth, but the growth does not stop but tends to some limited limit. The logarithmic trend, like the hyperbolic trend, represents a gradually decreasing process of changes. The exponential trend (Fig.7) corresponds to processes developing in an environment that does not create any limits for the growth of levels.

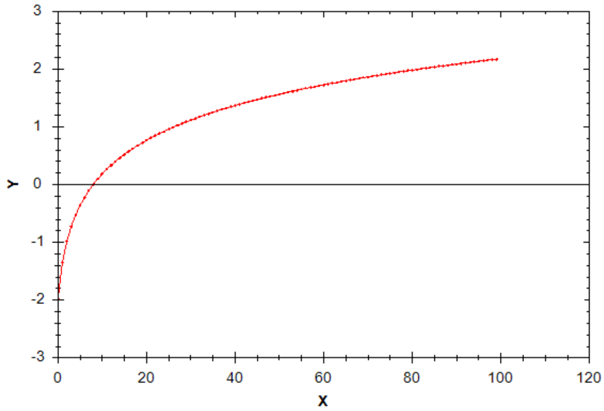

Figure 6: Logarithmic trend

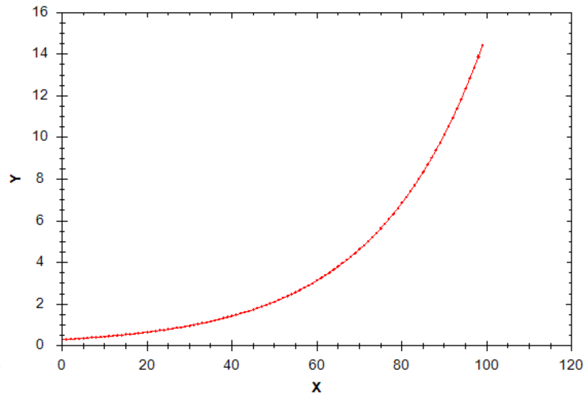

Figure 7: Exponential trend

5. Experiment description

The main idea of obtaining the trend wavelet test was that at each next level of time series decomposition the size of its approximation and details components decreases by half. Thus, at the last level of decomposition, the approximation component contains only two elements. Fig. 8 shows the decomposition of time series with a length of 32 values, obtained by formulas (2). At level 4 the reconstructed time series containing a trend has a clearly expressed difference in values.

The greater the amplitude of the trend, then the difference in values is greater. Fig. 9 shows the approximation components of the last level of time series decomposition with the trend (dashed line) and without trend (solid line). Numerical studies have shown that the Haar wavelet, due to its shape, allows to obtain a better result and to detect the presence of a trend.

Thus, as a test for the presence or absence of a trend, it is advisable to choose a random value

$$K(S,N,Haar\ wavelet) = abs(A1 - A2),$$

where *A1* and *A2* are values of the approximation component of the time series at the maximal level of decomposition.

In order to use the proposed test with the aim to accept or reject the hypothesis that the time series does not contain a trend, i.e. represents independent values of a normal random variable, it is necessary to investigate the random variable $K(S,N,Haar\ wavelet)$ for different types of trend and length of time series. The functions presented in Table 1 were selected as trend components. Figure 10 shows a model time series $S(t) = T(t) + \xi(t)$.

6. The experiment results and discussion

During the research, a sample of time series of 10,000 values were simulated for each type of trend. The numerical experiment showed that the value $K(S,N,Haarwavelet)$ has a normal distribution with zero mathematical expectation and mean square deviation depending on the length of the time series. This allows calculating the region of acceptance easily enough. If a sample value is in the interval $\left[K\left(S,N,Haarwavelet\right)_{N;1-\frac{\alpha}{2}}\right] < K_{sample} < \left[K\left(S,N,Haarwavelet\right)_{N;\frac{\alpha}{2}}\right]$, then the null hypothesis H_0 is accepted with a significance level α. Otherwise, the hypothesis is rejected (Fig. 11). Table 2 shows the values of the regions of acceptance for different lengths of time series. To carry out a comparative analysis, the following tests of trend presence were also considered: series test, inversion test, extremum test Spearman rank correlation, Foster-Stewart test, Fisher test, and Student's test [3,24,25].

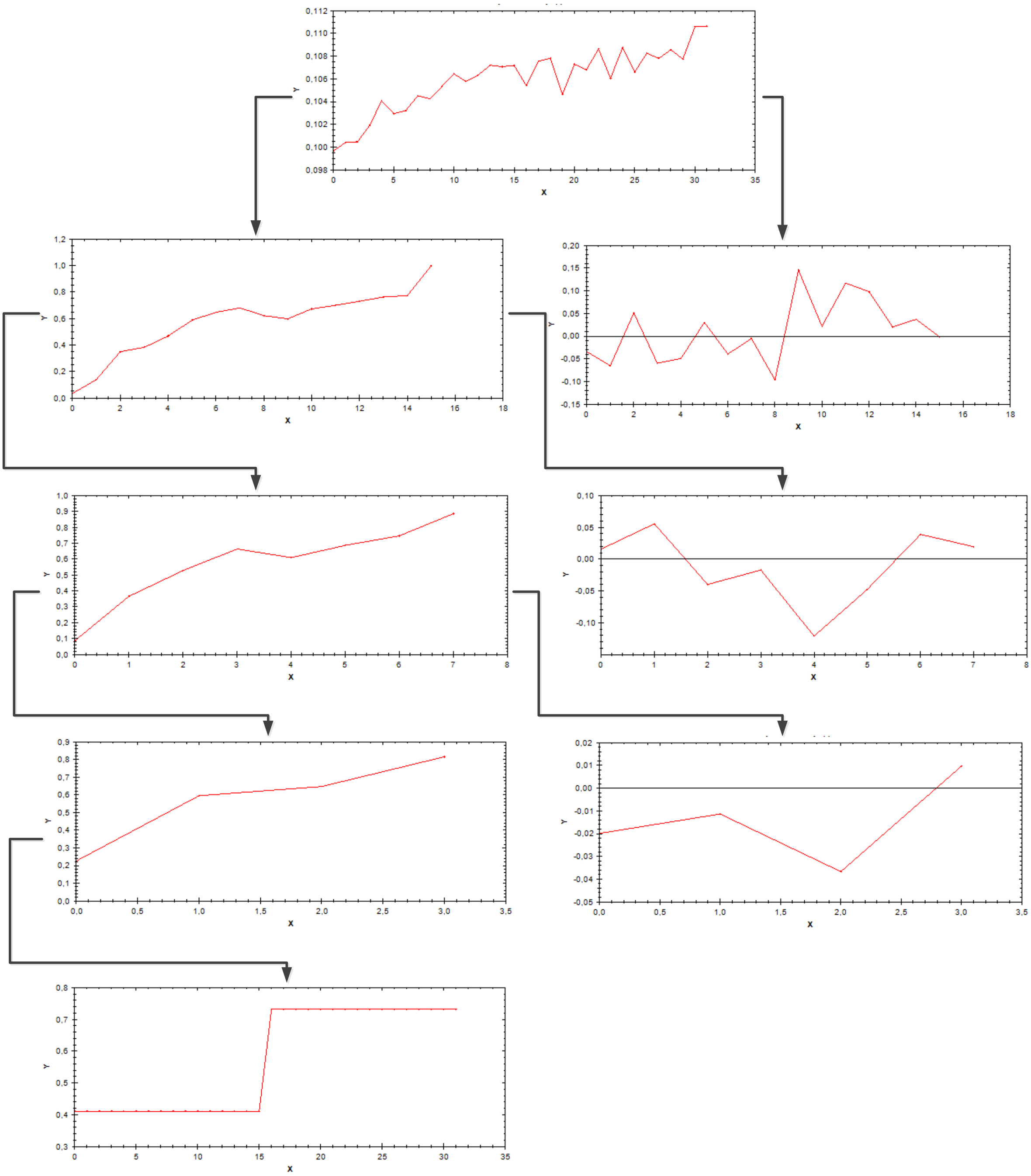

Figure 8: Decomposition of the time series using the Haar wavelet

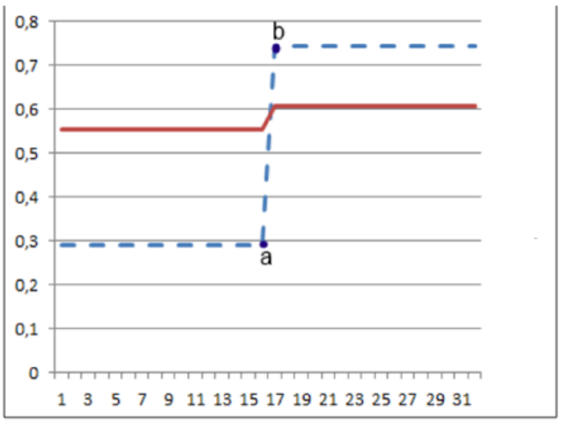

Figure 9: Approximation components with the trend (dashed line) and without trend (solid line).

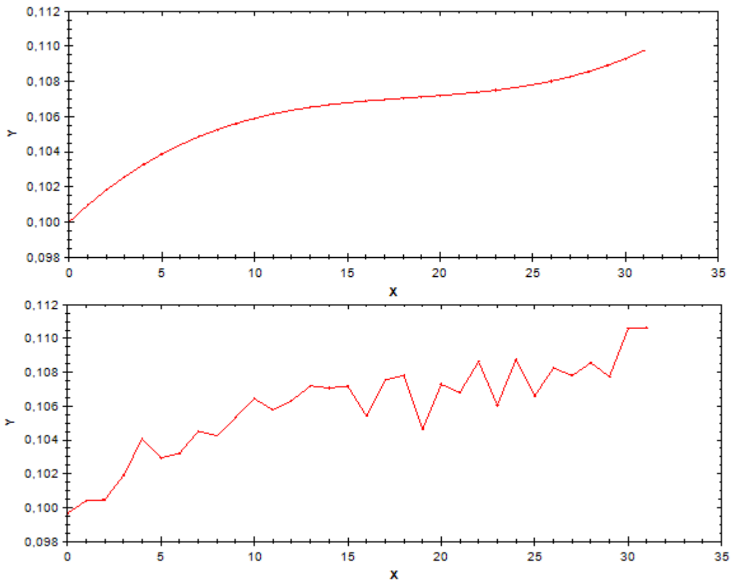

Figure 10: Model trend (top) and time series with trend (bottom)

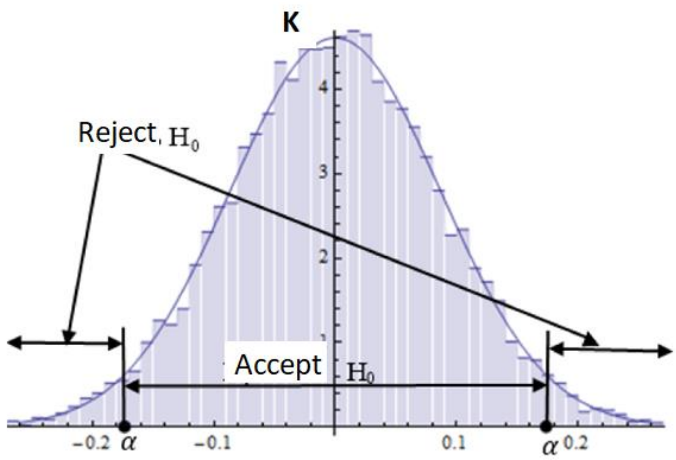

Figure 11: Sample distribution density $K(S, N, Haar\ wavelet)$

Table 2
Regions of Acceptance

Number of time series values	Regions of acceptance with α=0.05
N=8	$-0{,}482 \leq K_{sample} \leq +0{,}482$
N=16	$-0{,}2835 \leq K_{sample} \leq +0{,}2835$
N=32	$-0{,}17318 \leq K_{sample} \leq +0{,}17318$

A numerical experiment similar to the one described above was carried out and the best results in identifying the trend were obtained using the series test. A series is a sequence of observations of the same type, before and after which the observations of the opposite type are followed. The number of series S appearing in the sequence of observations of length N will have a certain sampling distribution. To test with significance level α, it is necessary to compare the observed value of the number of series with the limits of the regions of acceptance $S_{N;1-\frac{\alpha}{2}}$ and $S_{N;\frac{\alpha}{2}}$.

Table 3 shows the probabilities of type II error, i.e. deciding that the time series does not contain a trend when indeed there is a trend. The results are presented for the wavelet test and the series test. The probabilities were calculated for a sample of 10,000 values and the significance level $\alpha = 0.5$. The conducted numerical experiment showed the advantage of the proposed test over the many known statistical ones, especially when identifying the trend in the time series of up to 10 values. It should be noted that the worst result in both cases is the detection of a hyperbolic trend. This is explained by the fact that with this trend (Fig. 5), most of the time series is close to the asymptotic value.

Table 3
Probabilities of type II error

Trend	Wavelet test	Series test	Wavelet test	Series test	Wavelet test	Series test
	8 values		16 values		32 values	
Linear	0.22	0.98	0.005	0.10	0	0.01
Polynomial	0.24	0.99	0.01	0.18	0	0.14
Exponential	0.23	0.99	0.005	0.10	0	0.01
Hyperbolic	0.83	1	0.70	0.62	0.06	0.67
Power	0.25	0.99	0.05	0.10	0	0.01
Logarithmic	0.45	0.98	0.05	0.26	0	0.09

7. Conclusion

In this paper, we proposed a trend test based on the wavelet decomposition of the time series using the Haar wavelet function. It was shown that the test values have a normal distribution. The regions of acceptance about the absence of a trend for different lengths of time series have been calculated. It is shown that the proposed test allows to detection of the presence of a trend in time series of small length, starting from 8 values. Numerical studies have shown that the proposed test applies to the majority of trend functions, but is poorly suited to identify trends whose functions tend to the asymptotic value. Numerical comparative analysis with the series test was performed. The results indicate the advantages of the wavelet test for trend detection in time series of up to 30 values.

The results demonstrate the possibility to use the sample value of the criterion as one of the features for the classification or clustering of time series. Future research will focus on this direction.

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