Some Ways of Counteracting Possible Manipulations Within the AHP on The Base of Weighted Linear Equations

Oleksiy Oletsky [1] and Dmytro Dosyn [2]

[1] National University of Kyiv-Mohyla Academy, Skovorody St.,2, Kyiv, 04070, Ukraine
[2] Information Systems and Network Department, Institute of Computer Science and Information Technologies, Lviv Polytechnic National University (LPNU), Ukraine

Abstract

The paper considers a problem of enhancing the quality of pairwise comparisons within the Analytic Hierarchy Process (AHP). A situation when an expert, who is accountable for providing judgments in the form of pairwise comparisons, is not a person of a good integrity. They want to boost up a certain alternative but don't want to claim its advantage explicitly. Then applying procedures for rectifying inconsistency definitely shall result in order violations, and the system of automated decision making should decide what to do with that. An approach based on weighted systems of linear algebraic equations is considered. Some ways of choosing appropriate weights for counteracting manipulations are suggested.

Keywords [1]

Analytic Hierarchy Process, pairwise comparisons, improving consistency, weighted logarithmic least square method, order violation, counteracting manipulations

1. Introduction

Systems and tools for automated algorithmic-driven decision making are widely proliferated now. A very significant place among the algorithms applied in these systems is taken by the Analytic Hierarchy Process (AHP) suggested by T.Saaty [1-6 et al.]. AHP typically considers different connected levels of hierarchy, each of them involves estimations given by experts – usually in the form of pairwise comparisons between given alternatives. But there are many problems related to improving initial pairwise comparisons even if there is only one level of hierarchy.

Let there be n alternatives making a set $A = \{a_1, \dots, a_n\}$, and let M be a pairwise comparison matrix (PCM) provided by an expert. As a matter of fact, the matrix M represents some relation of preference. Namely, $m_{ij} > 1$ if $a_i \succ a_j$, and $m_{ij} = 1$ if $a_i \sim a_j$. Numerical values $u_i = u(a_i)$ representing measures of importance for each alternative can be obtained as the components of the Perronian vector, denoted as $y = \mathcal{Y}(M)$, that is the normalized eigenvector of M. Typically, $u(a_i) = y_i$. The other widely used way to get the importance of each alternative is to calculate the geometric mean of the corresponding row. The most popular scale of preference being used for comparing alternatives is probably the original Saaty scale presented in [1] (*1* stands for the parity of two compared alternatives, the values *2, 3, …,9* are the sequential grades of preference). Many other scales and approaches to building such scales have been suggested [6-11 et al.].

The logarithmic form of PCMs [12] is widely applied. A matrix C is a logarithmic form of the PCM M if its elements are as follows:

$$c_{ij} = \log_\tau m_{ij}, i, j = \overline{1, n}$$

where τ is a chosen logarithm base. In this paper there is no reason for stipulating any specific values for τ though this question might matter in some other contexts. What is really essential is that transforming the initial PCM to the logarithmic form allows to get additive pairwise comparisons [6].

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oletsky@ukma.edu.ua (O. Oletsky); dmytro.h.dosyn@lpnu.ua (D.Dosyn)
ORCID: 0000-0002-0553-5915 (O.Oletsky); 0000-0003-4040-4467 (D.Dosyn)

It is very helpful for analyzing consistency and for developing procedures for improving it, which is the main point of the paper.

The great problem is that initial PCMs directly provided by experts may be not of sufficient quality. Classical concepts of cardinal and ordinal consistency are ubiquitous, a lot of different indicators of inconsistency as well as plenty of approaches for rectifying inconsistency have been suggested [1, 6, 13-24 et al.], many of them are frankly heuristic. Since Saaty had suggested his famous inconsistency threshold (numbering *0.1*) that has been permanently debated and reviewed (one well-known recommendation is that this threshold should be reduced to *0.05*). Anyway, such approaches may yield good results in normal situations but not be so good if things are not that normal. Many studies are focused on the problem of mere improving consistency which often is confused with the problem of quality, whereas those problems are not the same. First of all, for an arbitrary square positive matrix claimed to be a PCM (even if it is generated randomly and moreover even if it is not an inverse-symmetric, in other terms reciprocal, matrix) we can easily construct the ideally consistent matrix with the same Perronian vector [e.g., 6]. If a procedure for rectifying inconsistency came to such a matrix with preserving the initial disorder, such a result obviously would not be good.

It appears very important to make plausible assumptions about possible sources of inconsistency. It may result from various "benign" factors such as objective difficulty with estimating alternatives, lack of the experts' awareness, errors caused by overlooks, inaccuracy, distraction etc. But there may be factors of another sort, which can be characterized as "malignant". An expert, who is accountable for providing judgments in the form of pairwise comparisons, may be not a person of good integrity. Suppose they want to boost up a certain alternative which is doubted to be the best one, but don't want to claim its advantage explicitly. Then, being aware of the algorithms for decision making implemented in the given system, they likely shall manipulate with pairwise comparisons to deceive the algorithms and force them to make an improper resolution. In addition to this, a board responsible for organizing the process of decision making may be not of sufficient integrity as well. They can, for example, involve dummy technical alternatives, impose irrelevant criteria etc. Such strategies are integrally related to situations of so-called order violations.

An order violation is a situation when $u(a_i) < u(a_j)$ whereas $a_i \succ a_j$. Order violations are ultimately inevitable if the relation represented by a PCM M is non-transitive, but such situations may occur for transitive relations as well and do so even if the given PCM is comparatively consistent.

Many techniques for enhancing consistency, being applied to an initial PCM which is ordinally but not of sufficient cardinal consistency and features an order violation, shall result in getting another PCM, which will be free of this flaw. A resulting PCM may be very consistent and sometimes be in a good accordance with the initial Perronian vector, this will be showcased below. Such a result may be good for "benign" situations (though may be not). But if there is a manipulation, it's just a false improvement of the given PCM. It is exactly what the manipulator wanted, which is to let algorithms make a judgment desirable for the manipulator despite their tricky estimations seemingly contradicting to that. And then a blame of an improper decision may be put on designers of algorithms but not on the manipulator. Technically, in such a situation when an expert states in their PCM that A is better than B but wants the algorithms to decide that B is better than A, just an order violation, which is an intentional order violation, shall be an integral part and a main goal of the whole manipulation. If such a foul play really took place, and an order violation has been detected, the latter may be considered as a telltale sign of a manipulation – but that might be not that case, that could be merely an accidental mistake.

Basically, there may be an opportunity to consult other experts and to form an average PCM based on many estimations. But sometimes this may be impossible. In addition to this, experts' opinions may be not independent, and/or they may be biased because of a common influence. So, we are still regarding the "pure" case when there is only one expert.

In this paper some ways to combining techniques for enhancing consistency of initial PCMs with counteracting possible manipulations resulting to order violations have been suggested and discussed.

2. Some examples of deliberate order violations

Firstly, we are going to use the standard Saaty scale. Let there actually be a competition between two alternatives: *A1* and *A2*.

Example 1

The first example is a very basic and well-known one, but we will consider its parametrical form. Suppose an expert wants to boost up the alternative *A2,* but they don't want to state that explicitly. Then they might try to manipulate by imposing a technical, obviously a worse alternative *A3* and providing a pairwise comparison matrix (PCM) as follows: *A1* gets a slight (quantified as q_1) preference over *A2* and *A3*, and *A2* gets more significant preference q_2 over *A3*. Let's denote such a parametrized PCM as $M^0(q_1,\ q_2)$, then

$$M^0(q_1, q_2) = \begin{pmatrix} 1 & q_1 & q_1 \\ \frac{1}{q_1} & 1 & q_2 \\ \frac{1}{q_1} & \frac{1}{q_2} & 1 \end{pmatrix}$$

The bad news for the manipulator is that within the standard Saaty scale *A2* wins if only $q_1 = 2, q_2 = 9$. Then the PCM takes a view

$$M^0(2, 9) = \begin{pmatrix} 1 & 2 & 2 \\ \frac{1}{2} & 1 & 9 \\ \frac{1}{2} & \frac{1}{9} & 1 \end{pmatrix}$$

Its Perronian vector approximately equals

$$(0.4385, \qquad 0.4561, \quad 0.1054)$$

Indeed. in this case *A2* wins. But another problem is that such a PCM must be very inconsistent. Really, its consistency index approximately equals

$$CI(M^0(2, 9)) \approx 0.2804$$

Since we are applying the standard Saaty scale, we can calculate the consistency ratio as well. It equals

$$CR\big(M^0(2, 9)\big) = \frac{CI(M^0(2, 9))}{RI(3)} \approx \frac{0.2804}{0.58} = 0.4834$$

One can recognize a PCM having such a consistency ratio as an unacceptable one.

To conceal their manipulations, experts and organizers might try to impose some other base for decision making, for instance as in the following example.

Example 2

The number of technical alternatives might be increased. Let an overall number of alternatives be *n*, among which only *A1* and *A2* be the real competitors, and the other *n-2* alternatives be the technical ones. Like the previous example, *A1* gets the slight preference q_1 over *A2* and the technical alternatives, *A2* gets more significant preference q_2 over the technical alternatives, and the other (technical) alternatives are on a par. Then the resulting parametrized PCM takes the view

$$M'(n, q_1, q_2) = \begin{pmatrix} 1 & q_1 & \dots & & \\ \frac{1}{q_1} & 1 & q_2 & \dots & \\ \dots & & 1 & 1 & \dots \end{pmatrix}$$

The performed experiments show that such a PCM ensures a victory for A2 just already if $n = 6, q_1 = 2, q_2 = 3$. In this case the Perronian vector approximately equals

$$(0.2801, \quad 0.2825, \quad 0.1094, \quad 0.1094, \quad 0.1094, \quad 0.1094)$$

The alternative *A2* wins, and the consistency index and ratio are as follows:

$$CI\big(M'(6,2,3)\big) \approx 0.0282,$$
$$CR\big(M'(6,2,3)\big) \approx 0.0228$$

These values of consistency are seemingly good, but the quality of the matrix is not. Anyway, the planned deliberate order violation caused by the manipulation with the provided PCM is expected.

The situation is even aggravated if so-called transitive scales are being applied [8 et al.] (such kind of scales stipulates that the following grade of preference is quantified τ times larger than the previous one, τ is a certain parameter). Then for defining preferences between *i*-th and *j*-th alternatives we can merely specify the value c_{ij} which is the distance between those alternatives in terms of grades. So, the parametrized PCM can be written as follows:

$$M^T(C,\tau) = \begin{pmatrix} 1 & \tau^{c_{12}} & \dots & \dots \\ \tau^{-c_{12}} & 1 & \tau^{c_{23}} & \dots \\ \dots & & & \end{pmatrix}$$

<u>Example 3</u>

Let *n=3*, that is there are three alternatives, and the situation is like that in Example 1.

Experiments carried out, for instance, in [25] show that the deliberate order violation is gained if

$$C = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 4 \\ -1 & -4 & 0 \end{pmatrix}$$

or if c_{23} is larger than 4.

For $\tau = 1.2$, the Perronian vector approximately equals

$$(0.3682, \quad 0.3912, \quad 0.2406)$$

The index of consistency approximately equals *0.0296*. Though applying such indices to transitive scales with different parameters is a special and not very clear issue, it should be the less the better anyway. The examples given above illustrate deliberate order violation means that what an expert wants to achieve is different from what they claim when constructing the PCM. Of course, it would be reasonable to ask the expert for explaining the situation. But the process of decision making can be very automated, and such a facility can be unavailable. Therefore, it appears important to develop algorithms and automated procedures aimed at detecting possible manipulations and counteracting them, or in other words, those robust to manipulations. To say it more accurately, detecting order violations is easy but counteracting is not.

3. A weighted system of linear equations

An approach to enhancing quality of pairwise comparisons based on solving systems of linear algebraic equations was considered in [25].

Let *M* be a pairwise comparison matrix provided by an expert, and *C* is its logarithmic form

Then the regarded system can be written in the following form:

$$WHx = Wb \tag{1}$$

where H and b are the matrix and the right side of the system of linear algebraic equations with respect to x_{ij}:

$$x_{ij} = c_{ij}, \ i = \overline{1,n}, j = \overline{\iota + 1, n} \tag{2}$$

$$\forall i, k > i, j > k \ \ x_{ik} + x_{kj} - x_{ij} = 0 \tag{3}$$

$$W = diag(w_i),$$

w_i are weighting coefficients which can be clearly interpreted as degrees of certainty about experts' estimations or of trust to them.

Basically, this system follows the idea of applying the logarithmic least square method for getting more consistent PCMs [6, 26, 27]. It focuses on analysis of triads and on minimizing distances between what is needed for cardinal consistency and what is really provided.

The system (1-3) is over-determined, it contains $n(n-1)/2$ unknowns and $\frac{n(n-1)}{2}+\frac{n(n-1)(n-2)}{6}$ equations. Equations forming the group (2) were named expert equations because they are reflecting experts' estimations. Equations of the group (3) were named consistency, or equidistant, equations, because they ensue from the requirements of cardinal consistency if those requirements are written in the logarithmic form. It's easy to show that the system has a single (pseudo)solution which can be obtained with the help of the Moore-Penrose pseudo-inversion [28].

The obtained x_{ij} shall form the new PCM. Such a process can be performed iteratively. The issue of convergence within iterative improvements has been studied in [29, 30]. It was shown that under certain conditions the consequence of PCMs converges to the ideally consistent PCM with zero consistency index, and this can be confirmed experimentally. But, as it was mentioned before, it is necessary to control the process so that a risk of transmitting a disorder in the initial data to final resolutions would be as low as possible.

As a tool for such a control we suggest using weighting coefficients for the equations in (1-3). The idea of weighting sources of information has been discussed, for instance, in [31] but this idea can be implemented in different ways. And the main problem is how to pick out the coefficients w_i .

Solving the unweighted system is not very helpful for counteracting intentional order violations. As it was mentioned before, in this case that can in fact enhance consistency of the PCM but hardly its quality. Experiments carried out in [24] confirm that such enhancements, which can be done once or iteratively, eventually result in the consistent PCM with the changed directions of preferences. Whereas in the initial matrix we had $A1 \succ A2$, in the resulting PCM we can get $A2 \succ A1$, and this is exactly the arranged order violation.

Let's illustrate this on the Example 3. Since a transitive scale is being used in that example, the pairwise comparison matrix *C* is presented in the logarithmic form from the very beginning.

For $\tau = 1.2$ its "classical" exponential form is

$$M = \begin{pmatrix} 1 & 1.2 & 1.2 \\ 0.8333 & 1 & 2.0736 \\ 0.8333 & 0.4823 & 1 \end{pmatrix}$$

The unweighted system (2-3) for this PCM takes a view

$$\begin{cases} x_{12} = 1 \\ x_{13} = 1 \\ x_{23} = 4 \\ x_{12} + x_{23} - x_{13} = 0 \end{cases}$$

Its solution yields the following updated PCM (in the exponential form)

$$M' = \begin{pmatrix} 1 & 1 & 1.44 \\ 1 & 1 & 1.7280 \\ 0.6944 & 0.5787 & 1 \end{pmatrix}$$

After 10 iterations we come to the matrix

$$M^{*} = \begin{pmatrix} 1 & 0.9410 & 1.5302 \\ 1.0627 & 1 & 1.6261 \\ 0.6535 & 0.6150 & 1 \end{pmatrix}$$

which is the approximate limit of consequent PCMs.

Its consistency index practically equals *0*. Its Perronian vector equals

$$(0.3682, \quad 0.3912, \quad 0.2406)$$

So, according to M^{*} $a_2 \succ a_1$, and $u(a_2) > u(a_1)$. There is no order violation now because it has been committed before, in the course of iterations. Yet, as we have mentioned before, this is not a satisfactory solution. So, such algorithms based on unweighted equations shall probably justify intentional order violations planned by experts who had manipulated. What algorithms of automated decision making should do is decide whether to accept an order violation resulting from their work or not. Let's now look at the problem how to find proper values for the weighting coefficients w_i.

4. Some ways of counteracting manipulations by changing weights of equations

Surely, algorithm-based counteracting manipulations can't be determined and straightforward even after an order violation has been detected. A system of decision making should have a set of rules aimed at considering a wide range of factors and principles, and this makes the problem rather complicated and intricated. We are going to discuss some heuristic rules of such a sort and look at how these rules may come in use for getting more or less appropriate coefficients in (1-3).

Hardly there is any reason for changing weights of consistency equations but changing those for expert equations appears promising. In this paper we are regarding the following heuristic rules:

- pairwise comparisons have a priority
- considering measures of inconsistency
- analyzing strongly connected components

Let's illustrate these rules one by one.

Pairwise comparisons have a priority

Meaningfully this rule has a following interpretation: if an expert explicitly stated in PCM that for the alternatives *A1*, *A2* $A1 \succ A2$, and there is no additional opportunity to consult them, then all transformations must remain this advantage and the relation $v(A1) > v(A2)$ should eventually hold. Unfortunately, following this rule may be problematic if there are multiple order violations or the given PCM is ordinally inconsistent at all. Technically, it surely can be applied for a single order violation. But this rule is too strict and can potentially cause problems if the detected order violation was an accidental error but not a deliberately planned result. Certainly, more attention should be paid to order violations between alternatives competing for an overall victory.

We are going to illustrate this rule on the Example 3. Let's entrench the preference $a_1 \succ a_2$ by taking weighting coefficients $w = (1.5,\ 1,\ 1,\ 1)$.

The iterative process described above leads to the consistent PCM

$$M^* = \begin{pmatrix} 1 & 1.051 & 1.6171 \\ 0.9515 & 1 & 1.5487 \\ 0.6184 & 0.6499 & 1 \end{pmatrix}$$

with the Perronian vector

$$(0.3891, \quad 0.3703, \quad 0.2406)$$

The problem of order violation has been eliminated, the first alternative wins.

Considering measures of inconsistency

The main idea is to detect which pairs *(i,j)* in the initial PCM *M* are the most inconsistent, that is for which pairs the values of deviations

$$d_{ij} = (m_{ij} - \frac{u(a_i)}{u(a_j)})^2 \tag{4}$$

are the largest.

Let's build the matrix $\mathcal{D}(M) = (d_{ij}, i, j = \overline{1, n})$ for the Example 3. It approximately equals

$$\mathcal{D}(M) = \begin{pmatrix} 0 & 0.0671 & 0.1091 \\ 0.0526 & 0 & 0.2002 \\ 0.0323 & 0.0176 & 0 \end{pmatrix}$$

As a matter of fact, only elements above the main diagonal in $\mathcal{D}(M)$ matter, since only they take part in forming the system (1-3). One can see that the largest value of deviation 0.2002 corresponds to the pair *(2, 3)*, and that is just a tricky pair which was the main instrument of the manipulation. If we decrease a weight of the equation corresponding to this pair, for example by taking $w = (1,\ 1,\ 0.6,\ 1)$, we will get the result of iterations which is the consistent matrix

$$M^* = \begin{pmatrix} 1 & 1.0301 & 1.3979 \\ 0.9708 & 1 & 1.3570 \\ 0.7154 & 0.7369 & 1 \end{pmatrix}$$

with the Perronian vector

$$(0.3723, \quad 0.3614, \quad 0.2663)$$

and with eliminated order violation.

But the regarded case was a very simple one, and the source of the manipulation has been distinctly indicated by the criteria of maximum deviation. In more tricky cases the situation may be more complicated.

Example 4

As a matter of fact, it is the Example 2 with the parameters $n = 6,\ q_1 = 2,\ q_2 = 3.$ The corresponding PCM equals (here the standard Saaty scale is applied)

$$M(6,2,3) = \begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 2 \\ \frac{1}{2} & 1 & 3 & 3 & 3 & 3 \\ \frac{1}{2} & \frac{1}{3} & 1 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{3} & 1 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{3} & 1 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{3} & 1 & 1 & 1 & 1 \end{pmatrix}$$

If we calculate deviations by the formula (4) for this matrix, the maximum deviation will be gained for the pair *(1, 2)*, which reflects the preference of a_1 over a_2. In such a situation decreasing weighting coefficient for the corresponding equation (for instance, by putting it to 0.75 whereas the other coefficients remain to equal 1) seems not to be helpful. The resulting Perronian vector approximately equals

$$(0.2531, \quad 0.2983, \quad 0.1122, \quad 0.1122, \quad 0.1122, \quad 0.1122)$$

which means that the advantage of the second alternative has been even increased.

In this case the manipulation was more concealed and less concentrated. Instead, increasing it according to the first heuristic rule (pairwise comparisons have a priority) yields much better results. For example, putting the coefficient to 1.25 yields the Perronian vector

$$(0.2859, \quad 0.2648, \quad 0.1123, \quad 0.1123, \quad 0.1123, \quad 0.1123)$$

and the first alternative wins.

Analyzing strongly connected components

Basically, picking out and analyzing strongly connected components (SCC) in the preference graph related to the given PCM appears to be quite useful if the ordinary consistency doesn't hold, that is the initial relation of preference is non-transitive. The idea is to build separate PCMs for each SCC and then to combine them with a specially constructed PCM connecting these SCCs.

Realizations of such an idea can be very different. For example, the following heuristic approach might be applied: for separate PCM $M^{(G_k)}$ within the *k*-th SCC denoted by G_k preferences could be got directly from the initial PCM *M*. But for making the procedure more flexible and adjustable we suggest that these coefficients should rather be calculated by the formula

$$m_{ij}^{(G_k)} = m_{ij}^{\rho_1},$$

$\rho_1 \leq 1$ is a smoothing coefficient designed to reduce a scatter of values within one SCC.

Preferences between SCCs are calculated by averaging with the additional treating. More technically, given the initial PCM *M*, the PCM $M^{(C)}$ for combining SCCs can be calculated by the following formula:

$$m_{kl}^{(c)} = r_{kl}^{\rho_2}$$

$$r_{kl} = \frac{\sum_{(i,j)\in B_{kl}} m_{ij}}{|B_{kl}|},$$

$$B_{kl} = \{(i,j): i \in G_k, \qquad j \in G_l\}$$

where G_k and G_l are the k-th and the l-th SCCs, $\rho_2 \geq 1$ is a sharpening coefficient designed to make difference between compared SCCs more distinct, and coefficients r_{kl} stand for relations between G_k and G_l.

Then the value of the i-th alternative can be calculated as follows:

$$u(a_i) = y_i^{(G_k)} \cdot u(G_k) \tag{5}$$

where $y_i^{(G_k)}$ should be obtained from the comparison matrix within the SCC G_k, and $u(G_k)$ is a value ascribed to G_k on the base of inter-CSS PCM $M^{(C)}$. Then the obtained vector of values should be normalized so that the sum of its components would equal 1.

In addition to this, we consider an extended preference graph which contains additional relations resulting from order violations. Let's try to apply this technique to the Example 4. The initial PCM is ordinally consistent. Formally, this means that each alternative constitutes a separate SCC with a single element. Since we have an order violation in the pair *(1, 2),* the arc *(2,1)* should be added to the extended graph. So, we have SCCs in the extended graph as follows:

$$G_1 = \{1,2\}, G_2 = \{3\}, G_3 = \{4\}, G_4 = \{5\}, G_5 = \{6\}$$

Comparisons within each SCC but G_1 are trivial and yield

$$y_1^{(G_k)} = 1, k = \overline{2,5}$$

It can be shown that taking $\rho_1 = \rho_2 = 1$ leads to not very good results. Let's take $\rho_1 = 0.5,\ \rho_2 = 1.5$. Then for G_1 we get the PCM

$$M^{(G_1)} = \begin{pmatrix} 1 & 2^{0.5} \\ \frac{1}{2^{0.5}} & 1 \end{pmatrix}$$

with the Perronian vector

$$(0.5858, \quad 0.4142)$$

Then we obtain the inter-CSS pairwise comparison matrix

$$\begin{pmatrix} 1 & 2.5^{1.5} & 2.5^{1.5} & 2.5^{1.5} & 2.5^{1.5} \\ \frac{1}{2.5^{1.5}} & 1 & 1 & 1 & 1 \\ \frac{1}{2.5^{1.5}} & 1 & 1 & 1 & 1 \\ \frac{1}{2.5^{1.5}} & 1 & 1 & 1 & 1 \\ \frac{1}{2.5^{1.5}} & 1 & 1 & 1 & 1 \end{pmatrix}$$

with the Perronian vector

$$(0.4970, \quad 0.1257, \quad 0.1257, \quad 0.1257, \quad 0.1257)$$

Finally, calculating by (5) and normalizing the resulting vector yields the following distribution of values among the alternatives:

$$(0.2912, \quad 0.2059, \quad 0.1257, \quad 0.1257, \quad 0.1257, \quad 0.1257)$$

The order violation has been eliminated.

5. Conclusions and discussion

In this paper the main attention is paid to possible manipulations with pairwise comparison matrices within the AHP-based decision making which can be deliberately committed by experts accountable for forming such judgments. The question is that a manipulator may want to boost up a certain alternative, but they don't want to be accused of non-integrity and to declare an advantage of that alternative explicitly. Then the manipulator would like to make up tricky PCMs, and what they want to achieve by doing that is force algorithms of decision making to reverse some of their judgments. This is the pure order violation, and it's the deliberate and planned one. Therefore, if a manipulation really takes place, it is typically accompanied with order violations. So, if an order violation is detected, it should be considered as a signal of possible manipulation though the violation might be caused by other, more benign reasons like overlooks, inaccuracy etc.

So, AHP-based algorithms for decision making should become more robust to possible manipulations and to acquire some techniques of tackling order violations with the aim of counteracting such manipulations. The strategy of dealing with order violations can't be easy and straightforward, it should be based on a set of parametrized rules and involve intelligent combining such rules.

An approach to enhancing consistency of pairwise comparisons on the base of iterative solving weighted systems of linear equations is being developed in the paper. An integral part of the suggested approach is to combine enhancing consistency itself with deciding what to do with detected order violations. Some rules aimed at picking out weights of these equations are regarded and illustrated in the paper. These rules are quite simple, other approaches certainly must exist. For instance, it seems promising to apply different techniques of reinforcement learning. Typically, the AHP-based decision making is more complicated than it was presented in the paper. It usually comprises some levels of hierarchy, at least one of them is related to criteria which possible decisions depend on. There is a vast room for foul plays with these criteria, and the issue how to make algorithms of automated decision making more robust deserves special research. It appears promising to combine the approach presented in this paper with the multi-level model "state-probability of choice" for multiagent decision making [32]. A game view on the problem is worth to be developed. This means considering a game in which a manipulator can try different strategies of achieving their goals, and a system of decision making should implement strategies of detecting manipulations and counteracting them.

Such considerations appear to be useful for many practical applications, such as financial privacy of the telecommunications space [33] or prioritizing cybersecurity measures using incomplete data [34].

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