Fuzzy Conceptual Knowledge Extraction and Retrieval Within Fuzzy Classes Decomposition

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Abstract

Extraction of hidden and nonobvious knowledge analyzing knowledge sources, as well as retrieval of required knowledge items within a knowledge base, are important features of modern intelligent knowledge-based systems. Therefore, in this paper, we propose the modification of the algorithm for the decomposition of fuzzy homogeneous classes of objects within fuzzy object-oriented dynamic networks, which allows the algorithm to perform knowledge retrieval within the set of semantically consistent subclasses constructed at the extraction stage, using the attribute-based and dependency-based filters. As the result, the modified algorithm reduces the knowledge search space avoiding the construction of semantically inconsistent subclasses and performing the filtration of semantically consistent ones depending on filtering parameters. To demonstrate the main application scenarios for the developed modification of the decomposition algorithm, we provided an illustrative example of the decomposition of a fuzzy homogeneous class of objects, using this modification.

Keywords [1]

Internal semantic dependencies, decomposition consistency, decomposition of fuzzy classes, knowledge extraction, knowledge retrieval.

1. Introduction

The extraction of hidden and nonobvious knowledge is one of the main tasks within the intelligent analysis of knowledge and knowledge engineering in general. Knowledge-based intelligent systems, which are equipped with the knowledge extraction module can extend the knowledge base by detecting and extracting new knowledge from knowledge sources. Another important knowledge management task is knowledge retrieval, which allows users of a system and the system itself to search and find required knowledge within the knowledge base. Both these tasks are connected and depending on chosen methods, knowledge extraction and knowledge retrieval can be implemented within a single module. Most of the known methods of knowledge extraction within object-oriented knowledge representation models are based on the two approaches. According to the first one, knowledge extraction is based on the logical implication around the concept hierarchies or concept lattices or inheritance relation between concepts. According to the second one, knowledge extraction is based on the usage of set-theoretical operations defined over the concept specifications, in particular difference and intersection. Such approaches are commonly used within the area of object-oriented programming, object-oriented databases, and ontologies. Despite all advantages of such approaches, they do not extract all hidden knowledge incorporated within such object-oriented representation structures as objects and classes. Therefore, in [18] we proposed another knowledge extraction approach, which is based on the decomposition of a fuzzy homogeneous class of objects on subclasses. It allows the extraction of subclasses, which cannot be obtained via reasoning over the conceptual hierarchies using the inheritance relation. Moreover, it provides an opportunity to organize knowledge retrieval during the extraction stage, using different techniques for the filtering of subclasses.

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In this paper, we consider the decomposition of fuzzy homogeneous classes of objects, within fuzzy object-oriented dynamic networks. We improved the decomposition algorithm, which was proposed in [18], to adapt it for more targetable knowledge retrieval using a filtering approach at the stage of subclasses construction. The proposed modification allows the decomposition algorithm to perform the knowledge extraction in a form of semantically consistent subclasses of a fuzzy homogenous class of objects, as well as knowledge retrieval within the set of all semantically consistent subclasses, using attribute and dependency filtering. We implemented attribute and dependency filtering of subclasses to perform the knowledge retrieval within a set of all semantically consistent subclasses of a fuzzy homogeneous class of objects, which the decomposition algorithm constructs at the knowledge extraction stage. To demonstrate how the improved algorithm reduces the knowledge search space and retrieves the required knowledge items, we provided an illustrative example of the decomposition of a particular fuzzy homogeneous class of objects. To show the possible application scenarios for proposed filtering parameters during the decomposition of fuzzy homogeneous classes of objects, we considered seven general possible configurations for these parameters, as well as the results of their usage. The rest of the paper has the following structure. Section 2 contains the main notions of fuzzy formal concept analysis, such as fuzzy context, fuzzy concept, and fuzzy concept lattice. Section 3 describes the analysis of a specification and a signature of a fuzzy homogeneous class of objects, and its internal semantic dependencies created by properties and methods. Section 4 presents the knowledge extraction via the decomposition of a fuzzy homogeneous class of objects on semantically consistent subclasses. Section 5 provides knowledge retrieval by selecting the subset of all semantically consistent subclasses using attribute-based and dependency-based filters. In the end, the conclusions section finishes the paper.

2. Fuzzy Formal Concept Analysis

Formal concept analysis is a powerful formal lattice-based framework for processing conceptual knowledge, proposed by Wille and Ganter [4, 5]. It provides means for the formal representation of domain knowledge in a form of formal contexts and defined within them formal concepts, which allows us to construct a concept lattice for a particular context and process corresponding conceptual hierarchy. Since many domains of knowledge, as well as knowledge itself, have a vague or imprecise nature, the formal concept analysis was generalized for fuzzy knowledge. It provides an opportunity to formalize such knowledge in terms of fuzzy formal context, associated with a particular domain, and then represent corresponding knowledge items related to the context as fuzzy formal concepts. After that, we can construct a fuzzy formal concept lattice, which consists of two isomorphic complete lattices, where one of them represents a fuzzy set of fuzzy objects, while another one represents a set of fuzzy attributes. Using the constructed fuzzy concept lattice, we can analyze it and extract new knowledge items, which previously were nonobvious or hidden.

Let us consider the main notions of fuzzy formal concept analysis, described in [1, 6, 8, 10-14]. The first fundamental notion is a fuzzy formal context, which combines the internal and external definitions of a class in object-oriented programming, object-oriented knowledge representation, object-oriented databases, etc.

Definition 1. *A fuzzy formal context is a triple* $K=(G,M,I)$, *where* G *is a set of objects,* M *is a set of attributes, and* $I=\varphi(G\times M)=\{(g,m)/\mu_I(g,m)\mid\forall g\in G,\ m\in M,\ \mu_I:G\times M\to[0,1]\}$ *is a fuzzy incidence relation* $G\times M$.

Using this definition, we also can represent a fuzzy formal context using a corresponding cross table, which describes the fuzzy relation I. The next fundamental notions are subsets of objects and attributes defined by the characteristic properties.

Definition 2. *A set of common attributes for all objects from a subset of objects* $A\subseteq G$ *is a set*

$$A^{\uparrow}=\{m\in M\mid\forall g\in A:\mu_I(g,m)\geq T\},$$

where T *is a confidence threshold.*

Definition 3. *A set of objects with a subset of common attributes* $B\subseteq M$ *is a set*

$$B^{\downarrow} = \{g \in G \mid \forall m \in B : \mu_I(g,m) \geq T\},$$

where T is a confidence threshold.

All previous notions form a background for the definition of the fuzzy formal concept, which provides an opportunity to formalize particular domain knowledge items in terms of fuzzy concepts.

Definition 4. *A fuzzy formal concept of the fuzzy formal context (G, M, I) with a confidence threshold T is a pair $(\varphi(A), B)$, where $A \subseteq G$ is an extent of the formal concept, while $B \subseteq M$ is its an intent, $A^{\uparrow} = B$, $B^{\downarrow} = A$, and $\varphi(A) = \{g, \mu_{\varphi(A)}(g) \mid \forall g \in A\}$, where a membership function $\mu_{\varphi(A)}(g)$ is defined as $\mu_{\varphi(A)}(g) = \min_{m \in B} \mu_I(g,m)$, where $\mu_I(g,m)$ is a membership value between object g and attribute m in I.*

The next fundamental notion is a fuzzy concept lattice, which defines a hierarchical structure over fuzzy formal concepts.

Definition 5. *A fuzzy concept lattice of a fuzzy formal context K with a confidence threshold T is a set $F(K)$ of all fuzzy concepts of K with a partial order $\leq$ and confidence threshold T.*

Analyzing the definitions above, we can see that by default formal context is defined using a set of attributes and a set of objects. Nevertheless, as was shown in [2, 3, 7], a formal context can be determined using a set of attributes and a set of classes. However, such application of fuzzy formal concept analysis has an important drawback, – it can produce inconsistent concepts. Algorithms for constructing concept lattice, described in [5], compute extents via the intersection of basic extents and do not consider the internal semantic dependencies of fuzzy homogeneous classes of objects. As the result, some of the generated fuzzy formal concepts will be semantically inconsistent, i.e. will contradict some internal semantic dependencies. Therefore, we used an alternative approach to the construction of the concept lattice proposed in [18].

3. Morphology of Fuzzy Classes

Each fuzzy homogeneous class of objects consists of a collection of properties (specification) and a collection of methods (signature), where the first one defines a structure for all fuzzy objects of the class, while the second one determines their behavior. As was shown in [18], there are some internal semantic dependencies among properties and methods of each fuzzy homogeneous class of objects, since some properties and/or methods can be defined using other properties and/or methods, as well as without using them. It is an important fact, that forms the background for the semantically consistent decomposition of a fuzzy class. Let us consider a particular fuzzy homogeneous class of objects, its internal semantic dependencies, and how they can affect the process of its decomposition.

However, first of all, let us consider the concept of a fuzzy homogeneous class of objects and its subclass within such knowledge representation model as fuzzy object-oriented dynamic networks (FOODNs), which was proposed in [15, 16] and later extended in [17].

Definition 6. *A fuzzy homogeneous class of objects is a collection*

$$T / M(T) = (P(T) / M(P(T)), F(T) / M(F(T))) / M(T) =$$
$$= ((p_1 / \mu(p_1), ..., p_n / \mu(p_n)) / M(P(T)), (f_1 / \mu(f_1), ..., f_m / \mu(f_m)) / M(F(T))) / M(T),$$

where $p_i \in P(T)$ is a crisp or fuzzy property of the class T, $f_i \in F(T)$ is its crisp or fuzzy method, $\mu(p_i): p_i(A) \to [0,1]$ and $\mu(f_j): f_j(A) \to [0,1]$ are measures of fuzziness of a property p_i and a method f_j, where A is an object of the class T, and $M(T)$ is a measure of fuzziness of the class T, defined in the following way

$$M(T) = (M(P(T)) + M(F(T)))/2 = (\mu(p_1) + ... + \mu(p_n) + \mu(f_1) + ... + \mu(f_m))/(n+m),$$
$$M(P(T)): P(T) \to (0,1],\ M(F(T)): F(T) \to (0,1],\ M(T): T \to (0,1].$$

Definition 7. *A fuzzy homogeneous class of objects $T_i / M(T_i)$ is a subclass of a fuzzy homogeneous class of objects $T / M(T)$, i.e. $T_i / M(T_i) \subseteq T / M(T)$, if and only if $P(T_i / M(T_i)) \subseteq P(T / M(T))$ and $F(T_i / M(T_i)) \subseteq F(T / M(T))$, where $P(T_i / M(T_i))$, $P(T / M(T))$, and $F(T_i / M(T_i))$, $F(T / M(T))$ are specifications and signatures of the class $T_i / M(T_i)$ and $T / M(T)$ respectively.*

To analyze the internal dependencies among properties and methods of fuzzy homogeneous classes of objects, let us consider particular examples of such classes. For this purpose, let us consider the fuzzy homogeneous class of objects Pt, which defines a concept of a fuzzy point on a plane, and has the following structure:

$$\begin{aligned}
Pt(&p_1 = (x, (v_1 \in V_x, \ldots))/1,\\
&p_2 = (y, (v_1 \in V_y, \ldots))/1,\\
&f_1 = get_x(pt, \ldots)/0.92,\\
&f_2 = get_y(pt, \ldots)/0.92\\
&)/0.96,
\end{aligned}$$

where $Pt.p_1/1$ and $Pt.p_2/1$ are fuzzy quantitative properties of the class $Pt/0.96$, which describe coordinates (x, y) of a point, and defined by the fuzzy sets V_x and V_y, where

$$V_x = \{w_i^- / \mu(w_i^-) + d_x / 1 + w_i^+ / \mu(w_i^+)\},\ V_y = \{q_j^- / \mu(q_j^-) + d_y / 1 + q_j^+ / \mu(q_j^+)\}$$

where $a_x < d_x < b_x$, $a_y < d_y < b_y$, and $d_x = (b_x - a_x)/2$, $d_y = (b_y - a_y)/2$, where $[a_x, b_x]$, $[a_y, b_y]$ are real numbers intervals, w_i^-, w_i^+, $i = \overline{1,...}$ as well as q_j^-, q_j^+, $j = \overline{1,...}$ are defined in the following way

$$w_i^- = d_x - k_x \cdot i,\ a_x < d_x - k_x \cdot i < d_x,\ w_i^+ = d_x + k_x \cdot i,\ d_x < d_x + k_x \cdot i < b_x,$$
$$q_j^- = d_y - k_y \cdot j,\ a_y < d_y - k_y \cdot j < d_y,\ q_j^+ = d_y + k_y \cdot j,\ d_y < d_y + k_y \cdot j < b_y,$$
$$\mu(w_i^-) = \frac{w_i^- - a_x}{d_x - a_x} - \delta_i^-,\ \delta_i^- = 1 - \mu(w_i^-) - \nu(w_i^-),\ \nu(w_i^-) = 1 - \mu(w_i^-),$$
$$\mu(w_i^+) = \frac{b_x - w_i^+}{b_x - d_x} - \delta_i^+,\ \delta_i^+ = 1 - \mu(w_i^+) - \nu(w_i^+),\ \nu(w_i^+) = 1 - \mu(w_i^+),$$
$$\mu(q_j^-) = \frac{q_j^- - a_y}{d_y - a_y} - \delta_j^-,\ \delta_j^- = 1 - \mu(q_j^-) - \nu(q_j^-),\ \nu(q_j^-) = 1 - \mu(q_j^-),$$
$$\mu(q_j^+) = \frac{b_y - q_j^+}{b_y - d_y} - \delta_j^+,\ \delta_j^+ = 1 - \mu(q_j^+) - \nu(q_j^+),\ \nu(q_j^+) = 1 - \mu(q_j^+),$$

where k_x and k_y are increments; $Pt.f_1/0.92$ and $Pt.f_2/0.92$ are fuzzy methods of the class $Pt/0.96$, which compute the defuzzification representation of the fuzzy qualitative properties $Pt.p_1/1$ and $Pt.p_2/1$, and are defined in the following way

$$f_1(pt) = \frac{\sum_{i=1}^{pt.x.v} \mu(pt.x.v) \cdot pt.x.v}{\sum_{i=1}^{pt.x.v} \mu(pt.x.v)},\quad f_2(pt) = \frac{\sum_{i=1}^{pt.y.v} \mu(pt.y.v) \cdot pt.y.v}{\sum_{i=1}^{pt.y.v} \mu(pt.y.v)},$$

where pt is a fuzzy object of the class $Pt/0.96$. The fuzzy homogeneous class of objects Pt has the measure of its fuzziness equal to 0.96, according to Definition 6.

Now let us consider another fuzzy homogeneous class of objects Rt, which defines a concept of a fuzzy rectangle, based on the class $Pt/0.96$, and has the following structure:

$$
\begin{aligned}
Rt(\,& p_1 = (vertex_1, (v, Pt))/1, \\
& p_2 = (vertex_2, (v, Pt))/1, \\
& p_3 = (vertex_3, (v, Pt))/1, \\
& p_4 = (vertex_4, (v, Pt))/1, \\
& p_5 = (is\_a\_rectangle, (vf_5(rt), v \in [0,1]))/0.93, \\
& f_1 = get\_vertex(rt, (i, \mathbb{R}^{+}), Pt)/0.9, \\
& f_2 = get\_side\_length(rt, (vertex_a, Pt), (vertex_b, Pt), \mathbb{R}^{+})/0.88, \\
& f_3 = compute\_perimeter(rt, \mathbb{R}^{+})/0.91, \\
& f_4 = compute\_area(rt, \mathbb{R}^{+})/0.85, \\
)&/0.94,
\end{aligned}
$$

where $Rt.p_1/1$, $Rt.p_2/1$, $Rt.p_3/1$, and $Rt.p_4/1$ are fuzzy quantitative properties, which describe vertices of a rectangle, and are defined as objects of the class of objects $Pt/0.96$; $Rt.p_5/0.93$ is a fuzzy qualitative property, which describes the satisfiability of basic rectangle properties for a rectangle object rt, (such as opposite sides of a figure should be parallel and all angles of a figure should be right) and is defined by the following verification function

$$vf_5(rt) : (rt.vertex_1, rt.vertex_2, rt.vertex_3, rt.vertex_4) \to [0,1],$$

$$
\begin{aligned}
vf_5 = ((rt.vertex_1.get\_x() = rt.vertex_2.get\_x()) \wedge (rt.vertex_3.get\_x() = rt.vertex_4.get\_x()) \wedge \\
\wedge (rt.vertex_1.get\_y() = rt.vertex_4.get\_y()) \wedge (rt.vertex_2.get\_y() = rt.vertex_4.get\_y()));
\end{aligned}
$$

$Rt.f_1/0.9$ is a fuzzy method, which returns $vertex_i$, $i \in \{1,2,3,4\}$ of a rectangle object rt in a form of objects of the class $Pt/0.96$, i.e. $f_1(rt,i) = (rt.vertex_i)$; $Rt.f_2/0.88$ is a fuzzy method, that computed a distance between two vertices of a rectangle object rt, i.e.

$$f_2(rt, vertex_a, vertex_b) = \sqrt{d_x - d_y},$$

$$d_x = (vertex_a.get\_x() - vertex_b.get\_x())^2,\; d_y = (vertex_a.get\_y() - vertex_b.get\_y())^2;$$

$Rt.f_3/0.91$ is a fuzzy method, which computes a perimeter of a rectangle object rt, and is defined in the following way $f_3(rt) = 2 \cdot (a+b)$, where

$$
\begin{aligned}
a = get\_side\_length(rt.get\_vertex(1), rt.get\_vertex(2)), \\
b = get\_side\_length(rt.get\_vertex(2), rt.get\_vertex(3));
\end{aligned}
$$

$Rt.f_4/0.85$ is a fuzzy method, which computes an area of a rectangle object rt, and is defined as follows $f_4(rt) = a \cdot b$, where

$$
\begin{aligned}
a = get\_side\_length(rt.get\_vertex(1), rt.get\_vertex(2)), \\
b = get\_side\_length(rt.get\_vertex(2), rt.get\_vertex(3)).
\end{aligned}
$$

The fuzzy homogeneous class of objects Rt has the measure of its fuzziness equal to 0.94, according to Definition 6. Now let us analyze the structure of the fuzzy homogeneous class of objects $Rt/0.94$ and detect all its internal semantic dependencies. According to [18], all internal semantic dependencies are divided into atoms and molecules, where both of them can be structural and functional. Therefore, let us consider their definitions in more detail.

Definition 8. *A fuzzy structural atom of a fuzzy homogeneous class of objects* $T / M(T)$ *is a singleton collection* $SA_i(T / M(T)) = \{T.p_i / \mu(T.p_i)\}$, *where* $T.p_i / \mu(T.p_i) \in P(T) / M(P(T))$ *is a crisp or fuzzy property defined without using any other properties and (or) methods of the class* $T / M(T)$, *where* $P(T) / M(P(T))$ *is its specification.*

Analyzing the specification of the fuzzy homogeneous class of objects $Rt / 0.94$, we can see that fuzzy quantitative properties $Rt.p_1 / 1$, $Rt.p_2 / 1$, $Rt.p_3 / 1$, and $Rt.p_4 / 1$, which describe a vertices of a fuzzy rectangle, are defined independently from all other properties and methods of the class, therefore they define corresponding fuzzy structural atoms within the class, i.e.

$$SA_1(Rt / 0.94) = \{Rt.p_1 / 1\},\ SA_2(Rt / 0.94) = \{Rt.p_2 / 1\},\ SA_3(Rt / 0.94) = \{Rt.p_3 / 1\},$$
$$SA_4(Rt / 0.94) = \{Rt.p_4 / 1\}.$$

Definition 9. *A fuzzy functional atom of a fuzzy homogeneous class of objects* $T / M(T)$ *is a singleton collection* $FA_i(T / M(T)) = \{T.f_i / \mu(T.f_i)\}$, *where* $T.f_i / \mu(T.f_i) \in F(T) / M(F(T))$ *is a crisp or fuzzy method defined without using any other properties and (or) methods of the class* $T / M(T)$, *where* $F(T) / M(F(T))$ *is its signature.*

Since fuzzy homogeneous class of objects $Rt / 0.94$ does not have any methods defined without the usage of any other properties or methods of the class, the set of internal semantic dependencies of the class does not contain fuzzy functional atoms.

Definition 10. *A fuzzy functional molecule of a fuzzy homogeneous class of objects* $T / M(T)$ *is a following collection* $FM_i(T / M(T)) = (T.f_i / \mu(T.f_i), \{T.x_{j_1} / \mu(T.x_{j_1}), \dots, T.x_{j_n} / \mu(T.x_{j_n})\})$, *where* $T.f_i / \mu(T.f_i) \in F(T) / M(F(T))$, $1 \le i \le |F(T) / M(F(T))|$ *is a crisp or fuzzy method defined based on the other methods and (or) properties*

$$T.x_{j_1} / \mu(T.x_{j_1}), \dots, T.x_{j_n} / \mu(T.x_{j_n}) \in P(T) / M(P(T)) \cup F(T) / M(F(T)),$$

which form fuzzy structural and (or) fuzzy functional atoms, and (or) are parts of smaller fuzzy molecules of the class $T / M(T)$, *where* $1 \le j_1 \le \dots \le j_n \le |P(T) / M(P(T)) \cup F(T) / M(F(T))|$ *and* $P(T) / M(P(T))$ *is a specification of the class* $T / M(T)$, *while* $F(T) / M(F(T))$ *is its signature.*

To analyze the specification and signature of the fuzzy homogeneous class of objects $Rt / 0.94$, we can observe that the fuzzy method $Rt.f_1 / 0.9$, which returns a vertex of a fuzzy rectangle according to its number, depends on the vertices themselves, as the result, it defines the following fuzzy functional molecule

$$FM_1(Rt / 0.94) = (Rt.f_1 / 0.9,\ \{Rt.p_1 / 1\},\ \{Rt.p_2 / 1\},\ \{Rt.p_3 / 1\},\ \{Rt.p_4 / 1\}).$$

The fuzzy method $Rt.f_2 / 0.88$, which computes a distance between two consequent vertices of a fuzzy rectangle, depends on the pair of such vertices and defines another fuzzy functional molecule

$$FM_2(Rt / 0.94) = (Rt.f_2 / 0.88,\ \{Rt.p_1 / 1, Rt.p_2 / 1\},\ \{Rt.p_2 / 1, Rt.p_3 / 1\},$$
$$\{Rt.p_3 / 1, Rt.p_4 / 1\},\ \{Rt.p_4 / 1, Rt.p_1 / 1\}).$$

And finally, fuzzy methods $Rt.f_3 / 0.91$ and $Rt.f_4 / 0.85$, which compute a perimeter and an area of a fuzzy rectangle, respectively, depend on three consequent vertices of the rectangle, the method for getting their coordinates, and the method for computing the length of rectangle sides, which are formed by vertices. As the result, both fuzzy methods define corresponding fuzzy functional molecules, i.e.

$$FM_3(Rt/0.94) = (Rt.f_3/0.91, \{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_1/1, Rt.p_2/1, Rt.p_3/1\},$$
$$\{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_2/1, Rt.p_3/1, Rt.p_4/1\},$$
$$\{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_3/1, Rt.p_4/1, Rt.p_1/1\},$$
$$\{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_4/1, Rt.p_1/1, Rt.p_2/1\}),$$

$$FM_4(Rt/0.94) = (Rt.f_4/0.85, \{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_1/1, Rt.p_2/1, Rt.p_3/1\},$$
$$\{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_2/1, Rt.p_3/1, Rt.p_4/1\},$$
$$\{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_3/1, Rt.p_4/1, Rt.p_1/1\},$$
$$\{Rt.f_2/0.88, Rt.f_1/0.9, Rt.p_4/1, Rt.p_1/1, Rt.p_2/1\}).$$

Definition 11. *A fuzzy structural molecule of a fuzzy homogeneous class of objects $T/M(T)$ is a following collection $SM_i(T/M(T)) = (T.p_i/\mu(T.p_i), \{T.x_{j_1}/\mu(T.x_{j_1}), \ldots, T.x_{j_n}/\mu(T.x_{j_n})\})$, where $T.p_i/\mu(T.p_i) \in P(T)/M(P(T))$, $1 \le i \le |P(T)/M(P(T))|$ is a crisp or fuzzy property defined based on the other properties and (or) methods*

$$T.x_{j_1}/\mu(T.x_{j_1}), \ldots, T.x_{j_n}/\mu(T.x_{j_n}) \in P(T)/M(P(T)) \cup F(T)/M(F(T)),$$

which form fuzzy structural and (or) fuzzy functional atoms, and (or) are parts of smaller fuzzy molecules of the class $T/M(T)$, where $1 \le j_1 \le \ldots \le j_n \le |P(T)/M(P(T)) \cup F(T)/M(F(T))|$ and $P(T)/M(P(T))$ is a specification of the class $T/M(T)$, while $F(T)/M(F(T))$ is its signature.

Analyzing the specification and signature of the fuzzy homogeneous class of objects $Rt/0.94$, we can observe that fuzzy qualitative property $Rt.p_5/0.93$, which describes the satisfiability of basic rectangle properties, and guarantees that points, considered as vertices of a fuzzy rectangle, exactly form the rectangle, is dependent on the all vertices of the figure and defines the following fuzzy structural molecule

$$SM_1(Rt/0.94) = (Rt.p_5/0.93, \{Rt.p_1/1, Rt.p_2/1, Rt.p_3/1, Rt.p_4/1\}).$$

All detected atoms and molecules of the class $Rt/0.94$ define a set of internal semantic dependencies, which can be determined in the following way.

Definition 12. *Internal semantic dependencies of a fuzzy homogeneous class of objects $T/M(T)$, which defines a fuzzy type t, is a set of fuzzy structural and functional atoms and fuzzy structural and functional molecules of the class $T/M(T)$, i.e.*

$$ISD(T/M(T)) = \{SA_1(T/M(T)), \ldots, SA_n(T/M(T)),\ FA_1(T/M(T)), \ldots, FA_m(T/M(T)),$$
$$SM_1(T/M(T)), \ldots, SM_w(T/M(T)),\ FM_1(T/M(T)), \ldots, FM_q(T/M(T))\},$$

where $SA_{i_1}(T/M(T))$, $i_1 = \overline{1,n}$ and $FA_{j_1}(T/M(T))$, $j_1 = \overline{1,m}$ are fuzzy structural and functional atoms of the fuzzy class $T/M(T)$, while $SM_{i_2}(T/M(T))$, $i_2 = \overline{1,w}$ and $FM_{j_2}(T/M(T))$, $j_2 = \overline{1,q}$ are its fuzzy structural and functional molecules respectively.

Using this definition, we can conclude that internal semantic dependencies of the fuzzy homogeneous class of objects $Rt/0.94$ can be represented as follows

$$ISD(Rt/0.94) = \{SA_1(Rt/0.94),\ SA_2(Rt/0.94),\ SA_3(Rt/0.94),\ SA_4(Rt/0.94),$$
$$FM_1(Rt/0.94),\ FM_2(Rt/0.94),\ FM_3(Rt/0.94),\ FM_4(Rt/0.94),\ SM_1(Rt/0.94)\}.$$

The set $ISD(Rt/0.94)$ forms the background for the decomposition of a fuzzy homogeneous class of objects $Rt/0.94$ on the semantically consistent subclasses.

4. Fuzzy Knowledge Extraction

Let us consider the definition of the decomposition of fuzzy homogeneous classes of objects based on the set of internal semantic dependencies, introduced in [18].

Definition 13. *Decomposition of fuzzy homogeneous class of objects* $T/M(T)$, *which defines a fuzzy type of object* t, *is a set of semantically consistent subclasses*

$$D(T/M(T)) = \{T_1/M(T_1) \subseteq T/M(T), \dots, T_n/M(T_n) \subseteq T/M(T)\},$$

where fuzzy homogeneous classes of objects $T_1/M(T_1), \dots, T_n/M(T_n)$ *do not contradict any fuzzy molecular internal semantic dependency* $d_i \in ISD(T/M(T))$, $1 \le i \le |T/M(T)|$ *of the class* $T/M(T)$.

The decomposition algorithm, which was proposed in [18], constructs subclasses of the class $T/M(T)$ solving correspondent constraint satisfaction problems, where internal semantic dependencies of the class are used as constraints to select only semantically consistent subclasses. Let us compute the full decomposition of the fuzzy homogeneous class of objects $Rt/0.94$, using the decomposition algorithm proposed in [18], with the following configuration:

$$D_F = (T/M(T) = Rt/0.94,\ C = ISD(Rt/0.94),\ N = [1,9],\ M = [0,1],\ \delta = 2).$$

As the result, the algorithm performed the decomposition of the class $T/M(T)$, and constructed the list of subclasses, which satisfy the decomposition configuration D_F. Therefore, we obtained four subclasses of the cardinality of 1, i.e.

$$SC_1^1(Rt)/1 = (p_1/1),\ SC_2^1(Rt)/1 = (p_2/1),\ SC_3^1(Rt)/1 = (p_3/1),\ SC_4^1(Rt)/1 = (p_4/1),$$

ten subclasses of the cardinality of 2, i.e.

$$SC_1^2(Rt)/1 = (p_1/1, p_2/1),\ SC_2^2(Rt)/1 = (p_1/1, p_3/1),\ SC_3^2(Rt)/1 = (p_2/1, p_3/1),$$
$$SC_4^2(Rt)/1 = (p_1/1, p_4/1),\ SC_5^2(Rt)/1 = (p_2/1, p_4/1),\ SC_6^2(Rt)/1 = (p_3/1, p_4/1),$$
$$SC_7^2(Rt)/0.95 = (p_1/1, f_1/0.9),\ SC_8^2(Rt)/0.95 = (p_2/1, f_1/0.9),$$
$$SC_9^2(Rt)/0.95 = (p_3/1, f_1/0.9),\ SC_{10}^2(Rt)/0.95 = (p_4/1, f_1/0.9),$$

fourteen subclasses of the cardinality of 3, i.e.

$$SC_1^3(Rt)/1 = (p_1/1, p_2/1, p_3/1),\ SC_2^3(Rt)/1 = (p_1/1, p_2/1, p_4/1),$$
$$SC_3^3(Rt)/1 = (p_1/1, p_3/1, p_4/1),\ SC_4^3(Rt)/1 = (p_2/1, p_3/1, p_4/1),$$
$$SC_5^3(Rt)/0.97 = (p_1/1, p_2/1, f_1/0.9),\ SC_6^3(Rt)/0.97 = (p_1/1, p_3/1, f_1/0.9),$$
$$SC_7^3(Rt)/0.97 = (p_2/1, p_3/1, f_1/0.9),\ SC_8^3(Rt)/0.97 = (p_1/1, p_4/1, f_1/0.9),$$
$$SC_9^3(Rt)/0.97 = (p_2/1, p_4/1, f_1/0.9),\ SC_{10}^3(Rt)/0.97 = (p_3/1, p_4/1, f_1/0.9),$$
$$SC_{11}^3(Rt)/0.96 = (p_1/1, p_2/1, f_2/0.88),\ SC_{12}^3(Rt)/0.96 = (p_2/1, p_3/1, f_2/0.88),$$
$$SC_{13}^3(Rt)/0.96 = (p_1/1, p_4/1, f_2/0.88),\ SC_{14}^3(Rt)/0.96 = (p_3/1, p_4/1, f_2/0.88),$$

thirteen subclasses of the cardinality of 4, i.e.

$$SC_1^4(Rt)/1 = (p_1/1, p_2/1, p_3/1, p_4/1),\ SC_2^4(Rt)/0.97 = (p_1/1, p_2/1, p_3/1, f_1/0.9),$$
$$SC_3^4(Rt)/0.97 = (p_1/1, p_2/1, p_4/1, f_1/0.9),\ SC_4^4(Rt)/0.97 = (p_1/1, p_3/1, p_4/1, f_1/0.9),$$
$$SC_5^4(Rt)/0.97 = (p_2/1, p_3/1, p_4/1, f_1/0.9),\ SC_6^4(Rt)/0.97 = (p_1/1, p_2/1, p_3/1, f_2/0.88),$$
$$SC_7^4(Rt)/0.97 = (p_1/1, p_2/1, p_4/1, f_2/0.88),\ SC_8^4(Rt)/0.97 = (p_1/1, p_3/1, p_4/1, f_2/0.88),$$
$$SC_9^4(Rt)/0.97 = (p_2/1, p_3/1, p_4/1, f_2/0.88),\ SC_{10}^4(Rt)/0.94 = (p_1/1, p_2/1, f_1/0.9, f_2/0.88),$$
$$SC_{11}^4(RT)/0.94 = (p_2/1, p_3/1, f_1/0.9, f_2/0.88),$$
$$SC_{12}^4(Rt)/0.94 = (p_1/1, p_4/1, f_1/0.9, f_2/0.88),$$
$$SC_{13}^4(Rt)/0.94 = (p_3/1, p_4/1, f_1/0.9, f_2/0.88),$$

seven subclasses of the cardinality of 5, i.e.

$$SC_1^5(Rt)/0.99 = (p_1/1, p_2/1, p_3/1, p_4/1, p_5/0.93),$$
$$SC_2^5(Rt)/0.98 = (p_1/1, p_2/1, p_3/1, p_4/1, f_1/0.9),$$
$$SC_3^5(Rt)/0.98 = (p_1/1, p_2/1, p_3/1, p_4/1, f_2/0.88),$$
$$SC_4^5(Rt)/0.96 = (p_1/1, p_2/1, p_3/1, f_1/0.9, f_2/0.88),$$
$$SC_5^5(Rt)/0.96 = (p_1/1, p_2/1, p_4/1, f_1/0.9, f_2/0.88),$$
$$SC_6^5(Rt)/0.96 = (p_1/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88),$$
$$SC_7^5(Rt)/0.96 = (p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88),$$

eleven subclasses of the cardinality of 6, i.e.

$$SC_1^6(Rt)/0.97 = (p_1/1, p_2/1, p_3/1, p_4/1, p_5/0.93, f_1/0.9),$$
$$SC_2^6(Rt)/0.97 = (p_1/1, p_2/1, p_3/1, p_4/1, p_5/0.93, f_2/0.88),$$
$$SC_3^6(Rt)/0.96 = (p_1/1, p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88),$$
$$SC_4^6(Rt)/0.95 = (p_1/1, p_2/1, p_3/1, f_1/0.9, f_2/0.88, f_3/0.91),$$
$$SC_5^6(Rt)/0.95 = (p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91),$$
$$SC_6^6(Rt)/0.95 = (p_1/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91),$$
$$SC_7^6(Rt)/0.95 = (p_1/1, p_2/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91),$$
$$SC_8^6(Rt)/0.94 = (p_1/1, p_2/1, p_3/1, f_1/0.9, f_2/0.88, f_4/0.85),$$
$$SC_9^6(Rt)/0.94 = (p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_4/0.85),$$
$$SC_{10}^6(Rt)/0.94 = (p_1/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_4/0.85),$$
$$SC_{11}^6(Rt)/0.94 = (p_1/1, p_2/1, p_4/1, f_1/0.9, f_2/0.88, f_4/0.85),$$

seven subclasses of the cardinality of 7, i.e.

$$SC_1^7(Rt)/0.96 = (p_1/1, p_2/1, p_3/1, p_4/1, p_5/0.93, f_1/0.9, f_2/0.88),$$
$$SC_2^7(Rt)/0.96 = (p_1/1, p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91),$$
$$SC_3^7(Rt)/0.95 = (p_1/1, p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_4/0.85),$$
$$SC_4^7(Rt)/0.93 = (p_1/1, p_2/1, p_3/1, f_1/0.9, f_2/0.88, f_3/0.91, f_4/0.85),$$
$$SC_5^7(Rt)/0.93 = (p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91, f_4/0.85),$$
$$SC_6^7(Rt)/0.93 = (p_1/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91, f_4/0.85),$$
$$SC_7^7(Rt)/0.93 = (p_1/1, p_2/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91, f_4/0.85),$$

and three subclasses of the cardinality of 8, i.e.

$$SC_1^8(Rt)/0.95 = (p_1/1, p_2/1, p_3/1, p_4/1, p_5/0.93, f_1/0.9, f_2/0.88, f_3/0.91),$$
$$SC_2^8(Rt)/0.94 = (p_1/1, p_2/1, p_3/1, p_4/1, p_5/0.93, f_1/0.9, f_2/0.88, f_4/0.85),$$
$$SC_3^8(Rt)/0.94 = (p_1/1, p_2/1, p_3/1, p_4/1, f_1/0.9, f_2/0.88, f_3/0.91, f_4/0.85).$$

Let us analyze the computed results of the full decomposition of the fuzzy homogeneous class of objects $Rt/0.94$ and compare them with the direct decomposition of the class, which includes all possible subclasses of the class $Rt/0.94$. As we can see, the class $Rt/0.94$ has five properties and four methods, that allow us to construct $2^9 = 512$ of its formally possible subclasses (i.e. a power set). Using the formula for the computation of binomial coefficients

$$C_n^k = \frac{n!}{k!(n-k)!},$$

where n is a number of properties and methods of the class, while k is the number of properties and methods of its particular subclass, we also can compute the number of subclasses of different cardinality. The results of these computations are represented in Table 1. The first line means cardinality of subclasses, and the second and the third lines represent quantities of all formally possible and all semantically consistent subclasses of certain cardinality. The fourth line contains decomposition consistency coefficients for subclasses of a particular cardinality, which was computed using the following formula

$$DC(T/M(T)) = \frac{|D(T/M(T))|}{|PS(T/M(T))|-2} \cdot 100\%,$$

where $D(T/M(T))$ is a set of all semantically consistent subclasses of the fuzzy homogeneous class of objects $T/M(T))$, while $PS(T/M(T))$ is a set of its all formally possible subclasses (a power set).

Table 1

Quantitative analysis of subclasses of the fuzzy homogeneous class of objects $Rt/0.94$

Cardinality	1	2	3	4	5	6	7	8	Total
Possible Subclasses	9	36	84	126	126	84	36	9	**510**
Consistent Subclasses	4	10	14	13	7	11	7	3	**69**
Decomposition Consistency	57%	29%	17%	10%	6%	13%	21%	43%	**14%**

Analyzing Table 1, we can see that the total decomposition consistency coefficient of the fuzzy homogeneous class of objects $Rt/0.94$ is approximately equal to 14%, which means that approximately 86% of all formally possible subclasses of the class are semantically inconsistent. Since the number of all formally possible subclasses of the fuzzy homogeneous class of objects grows exponentially depending on the number of properties and methods of the class, this fact allows us to avoid the extraction of semantically inconsistent knowledge and efficiently reduce the knowledge search space for the class $Rt/0.94$ approximately by 7 times, i.e. $100\%:14\% \approx 7$. In addition, such an approach avoids the production of semantically inconsistent concepts, which can be constructed by methods of fuzzy formal concept analysis, during the construction of a fuzzy concept lattice.

Using all data from Table 1, we constructed the subclass lattice tower of the fuzzy homogeneous class of objects $Rt/0.94$, which graphically represents the semantic consistency of the subclass lattice elements and allow us to estimate the knowledge extraction and search space from another perspective. Analyzing Figure 1, we can see figures, which have a form similar to a tower, they are towers of subclasses lattices. The highest tower is a tower of subclasses lattice of the class $Rt/0.94$. Yellow circles with numbers downside mean the cardinality of corresponding sequences of subclasses, where a number describes appropriate cardinality. Subclasses of the same cardinality form an antichain of a subclass lattice. Green circles with the gray border mean semantically consistent subclasses of the class $Rt/0.94$, while gray circles mean semantically inconsistent ones. To analyze the tower of subclasses lattices, we can use an interpretation, according to which the green circles are lighted sections or rooms of the tower, while gray circles are unlighted ones. The small green tower next to the highest tower is its modified version, which contains only semantically consistent subclasses. These two towers illustrate the knowledge search space reduction provided by the decomposition algorithm due to avoiding the construction of semantically inconsistent subclasses. Other green-gray towers of subclasses lattices, which are bordered by gray rectangles, represent the decomposition of particular subclasses of the class $Rt/0.94$, using the same algorithm. They show, that each non-empty subclass of the cardinality n, where $1 < n < |T/M(T)|$, can be also decomposed on the subclasses.

5. Fuzzy Knowledge Retrieval

As it was noted in [19, 20], the main goal of many retrieval algorithms is to reduce the search space as much as possible. One of the approaches to reducing the knowledge search space was proposed in [9], according to which, formal contexts can be matched by some of their sub-contexts the following relations: $\{gJm_s \mid \exists m \in m_s, gIm\}$, $\{gJm_s \mid \forall m \in m_s, gIm\}$, $\{gJm_s \mid (|m \in m_s, gIm| / |m_s|) \geq \alpha_{m_s}\}$. However, using such an approach produces additional concept lattices and requires matching them with the main concept lattice creating corresponding clusters, which can affect the knowledge extraction and retrieval performance. In the previous section, we used the algorithm for the decomposition of fuzzy homogeneous classes of objects, which was proposed in [18]. The main goal of the algorithm is to extract hidden or non-obvious knowledge in a form of semantically consistent subclasses of a fuzzy homogeneous class of objects, via its decomposition. It reduces the knowledge search space by avoiding the construction of semantically inconsistent subclasses of a fuzzy homogeneous class of objects during its decomposition. However, the algorithm uses parameters

$$N = [n_1, ..., n_k],\ 1 \le k \le |T / M(T)|,\ \ M = [a, b] \subseteq [0,1],$$

where N defines a sequence of required cardinalities for subclasses, which will be obtained as the result of the decomposition, while M determines required measure of their fuzziness. These parameters allow the algorithm not only to extract subclasses of a fuzzy homogeneous class of objects, using corresponding restrictions but also retrieve them using filtering. As the result, the algorithm constructs a subset of all semantically consistent sub-classes of a fuzzy homogeneous class of objects. Therefore, we can conclude that the decomposition algorithm performs knowledge extraction via the decomposition of a fuzzy homogeneous class of objects, as well as knowledge retrieval via the filtration of the set of semantically consistent subclasses.

Despite all advantages of the algorithm, it can be modified and improved in the context of knowledge retrieval by adding additional filtration parameters, which will provide an opportunity to filter the subclasses using attributes and dependencies. According to this, let us modify the decomposition algorithm by adding a parameter for filtering by attributes, i.e.

$$Q_a = [include = [T.a_{i_1} / \mu(a_{i_1}), ..., T.a_{i_w} / \mu(a_{i_w})],\ exclude = [T.a_{j_1} / \mu(a_{j_1}), ..., T.a_{j_q} / \mu(a_{j_q})]],$$

where $Q_a[include]$ and $Q_a[exclude]$ define a list of attributes (properties and/or methods) of a fuzzy homogeneous class of objects $T / M(T)$, which should be present and absent in all semantically consistent subclasses of the class, constructed by the algorithm, and where $1 \le i_1 \le ... \le i_w \le |T / M(T)|$, and $1 \le i_j \le ... \le j_q \le |T / M(T)|$.

Since any fuzzy homogeneous class of objects has its own internal semantic dependencies, they can be used as an additional filtering parameter. Therefore, let us modify the decomposition algorithm by adding the corresponding parameter for filtering by dependencies, i.e.

$$Q_d = [include = [d_{i_1}(T / M(T)), ..., d_{i_v}(T / M(T))],\ exclude = [d_{j_1}(T / M(T)), ..., d_{j_m}(T / M(T))]],$$

where $Q_d[include]$ and $Q_d[exclude]$ define a list of internal semantic dependencies (atoms and/or molecules) of a fuzzy homogeneous class of objects $T / M(T)$, which should be present and absent in all semantically consistent subclasses of the class, constructed by the algorithm, and where $1 \le i_1 \le ... \le i_v \le |ISD(T / M(T))|$, and $1 \le i_j \le ... \le j_m \le |ISD(T / M(T))|$.

Attribute and dependency filtering parameters allow the algorithm to reduce the number of constructed subclasses, as well as to find only required subclasses, among all semantically consistent ones, according to the specified query. Using these two parameters, we modified the decomposition algorithm proposed in [18], in the following way.

Algorithm 1. Decomposition of fuzzy homogeneous classes of objects.

Require: $T / M(T)$, C, N, M, δ, Q_a, Q_d

Ensure: D

```
D := {};
for n ∈ N do
    t := {};
    for i = 1, ..., 2^n − 1 do
        if binary(i).count(1) = i then
```

```
        for a_j / μ(a_j) ∈ T / M(T), j = 1,...,|T / M(T)| do
            if (i & (1 << j)) > 0 then
                t.add(a_j / μ(a_j));
            μ := compute_fuzziness(t, δ);
            if μ ∈ M then
                sutisfy := true;
                for all c ∈ C do
                    if not is_satisfy(t, c) then
                        satisfy := false;
                        break;
                if satisfy then
                    if satisfy_query(t, Q_a) and satisfy_query(t, Q_d) then
                        D.add(t / μ);
        t := {};
return D.
```

The modified algorithm decomposes a fuzzy homogeneous class of objects $T / M(T)$ constructing the subset of its subclasses, which are semantically consistent ones, i.e. do not contradict any internal semantic dependency $c \in C = ISD(T / M(T))$, and have a required cardinalities, measures of fuzziness, and satisfy attribute and dependency filters. The generation of semantically consistent subclasses of the class $T / M(T)$ is performed due to the resolving constraint satisfaction problems, which allows the algorithm to extract only consistent subclasses. The procedure $\text{is\_satisfy}(t,c)$ verifies that a particular candidate-subclass $t / M(t) \subseteq T / M(T)$ does not contradict a certain constraint $c \in ISD(T / M(T))$ in a form of internal semantic dependency (i.e. structural or functional molecule). If the candidate-subclass does not contradict the constraint, the procedure $\text{is\_satisfy}(t,c)$ returns *true*, in opposite case it returns *false*, and if the constraint is not applicable to the subclass, it returns *none*. The procedure $\text{is\_satisfy}(t,c)$ is invoked only for those candidate-subclasses, which have the appropriate cardinality and the measure of fuzziness defined by the parameter N, i.e. $|t / \mu(t)| \in N$, and the parameter M, i.e. $\mu(t) \in M$, respectively. It reduces the algorithm complexity, avoiding invocation of the procedure $\text{is\_satisfy}(t,c)$ for all candidate subclasses.

Procedure 1. $\text{compute\_fuzziness}(t,\delta)$

Input: T, δ
Output: $M(T) \in [0,1]$

```
sum := 0;
for a_i ∈ T, i = 1,...,|T| do
    sum := sum + μ(a_i);
M(T) := round(sum / max(|T|,1), δ);
return M(T).
```

Procedure 2. $\text{is\_satisfy}(t,c)$

Input: t, c
Output: satisfy $\in$ {**true**, **false**, **none**}

```
satisfy := none;
if c[0] ∈ t then
    satisfy := false;
    for c[i] ∈ c, i = 1,...,|c| do
        for c[i][j] ∈ c[i],
                j = 1,...,|c[i]| do
            if c[i][j] ∈ t then
                satisfy := true;
            else
                satisfy := false;
                break;
        if satisfy then
            return satisfy;
return satisfy.
```

Procedure 3. $\text{satisfy\_query}(t,Q)$

Input: t, Q

Output: satisfy $\in$ { **true**, **false** }

if $|Q[include]| = 0$ **and** $|Q[exclude]| = 0$ **then**
    return true;
for $q \in Q[include]$ **do**
    if $q \notin t$ **then**
        return false;
for $q \in Q[exclude]$ **do**
    if $q \in t$ **then**
        return false;
return true.

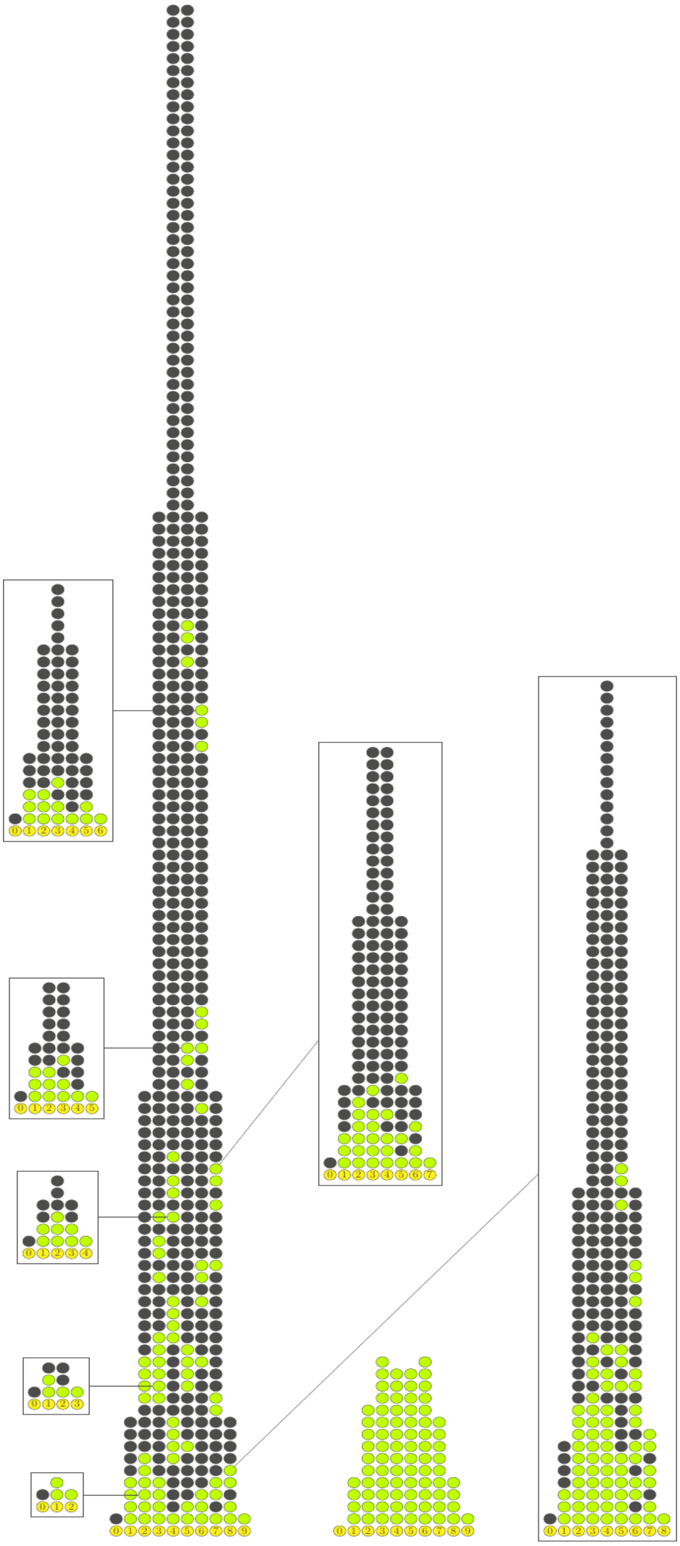

Figure 1: Tower of subclass lattice of the fuzzy homogeneous class of objects $Rt/0.94$

The second part of the filtration for selected semantically consistent subclasses, which satisfy all restrictions from set C, and have the required cardinality and measure of fuzziness, is performed by the procedure $\text{satisfy\_query}(t, Q)$. At this stage, from all semantically consistent subclasses previously selected, the algorithm retrieves only those subclasses which satisfy the attribute and dependency filters. If a particular subclass of the decomposed class $T / M(T)$ does not satisfy attribute and/or dependency filters, it will be excluded from the resulting decomposition. Such filtering is useful when we have additional meta-knowledge about the structure and/or behavior of semantically consistent subclasses of the class $T / M(T)$, which makes the retrieval process more targetable.

To demonstrate the particular applications of the proposed modification of the decomposition algorithm, let us consider a few examples of decomposition-based retrieval of fuzzy knowledge using the fuzzy homogeneous class of objects $Rt / 0.94$, which was described in section 2. Suppose we want to find all semantically consistent subclasses of the class $Rt / 0.94$, which have a cardinality of 5 or 6, a measure of fuzziness defined on the interval $[0.95,1]$, computed with the accuracy of 2 signs, and which contain attributes $Rt.p_1 / 1$ and $Rt.f_1 / 0.9$. This can be done by using Algorithm 1 with the following configuration:

$$D_1 = (Rt / 0.94,\ ISD(Rt / 0.94),\ N = [5,6],\ M = [0.95,1],$$
$$Q_a = [include = [Rt.p_1 / 1,\ Rt.f_1 / 0.9],\ exclude = []],\ Q_d = [include = [],\ eclude = []]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt / 0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_1:

$$D_1(Rt / 0.94) = \{SC_2^5(Rt) / 0.98,\ SC_4^5(Rt) / 0.96,\ SC_5^5(Rt) / 0.96,\ SC_6^5(Rt) / 0.96,$$
$$SC_1^6(Rt) / 0.97,\ SC_3^6(Rt) / 0.96,\ SC_4^6(Rt) / 0.95,\ SC_5^6(Rt) / 0.95,$$
$$SC_6^6(Rt) / 0.95,\ SC_8^6(Rt) / 0.94,\ SC_9^6(Rt) / 0.94,\ SC_{10}^6(Rt) / 0.94\}.$$

Let us consider another case. Suppose we want to find all semantically consistent subclasses of the class $Rt / 0.94$, which have a cardinality of 5 or 6, a measure of fuzziness defined on the interval $[0.95,1]$, computed with the accuracy of 2 signs, and which do not contain the attribute $Rt.f_2 / 0.88$. This can be done by using Algorithm 1 with the following configuration:

$$D_2 = (Rt / 0.94,\ ISD(Rt / 0.94),\ N = [5,6],\ M = [0.95,1],$$
$$Q_a = [include = [],\ exclude = [Rt.f_2 / 0.88]],\ Q_d = [include = [],\ exclude = []]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt / 0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_2:

$$D_2(Rt / 0.94) = \{SC_1^5(Rt) / 0.99,\ SC_2^5(Rt) / 0.98,\ SC_1^6(Rt) / 0.97\}.$$

Let us assume that we need to find all semantically consistent subclasses, which simultaneously satisfy configurations D_1 and D_2. This can be done by using Algorithm 1 with the following configuration:

$$D_3 = (Rt / 0.94,\ ISD(Rt / 0.94),\ N = [5,6],\ M = [0.95,1],$$
$$Q_a = [include = [Rt.p_1 / 1,\ Rt.f_1 / 0.9],\ exclude = [Rt.f_2 / 0.88]],$$
$$Q_d = [include = [],\ exclude = []]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt / 0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_3:

$$D_3(Rt / 0.94) = \{SC_2^5(Rt) / 0.98,\ SC_1^6(Rt) / 0.97\}.$$

Now let us assume that we need to find all semantically consistent subclasses of the class $Rt / 0.94$, which have a cardinality of 5 or 6, a measure of fuzziness defined on the interval $[0.95,1]$, computed with the accuracy of 2 signs, and which contain structural molecule $SM_1(Rt/0.94)$. This can be done by using Algorithm 1 with the following configuration:

$$D_4 = (Rt/0.94,\ ISD(Rt/0.94),\ N = [5,6],\ M = [0.95,1],\ Q_a = [include = [],\ exclude = []],$$
$$Q_d = [include = [SM_1(Rt/0.94)],\ exclude = []]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt/0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_4:

$$D_4(Rt/0.94) = \{SC_1^5(Rt)/0.99,\ SC_1^6(Rt)/0.97,\ SC_2^6(Rt)/0.97\}.$$

Let us consider another case. Suppose we need to find all semantically consistent subclasses of the class $Rt/0.94$, which have a cardinality of 5 or 6, a measure of fuzziness defined on the interval $[0.95,1]$, computed with the accuracy of 2 signs, and which do not contain the functional molecule $FM_2(Rt/0.94)$. This can be done by using Algorithm 1 with the following configuration:

$$D_5 = (Rt/0.94,\ ISD(Rt/0.94),\ N = [5,6],\ M = [0.95,1],\ Q_a = [include = [],\ exclude = []],$$
$$Q_d = [include = [],\ exclude = [FM_2(Rt.0,94)]]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt/0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_5:

$$D_5(Rt/0.94) = \{SC_1^5(Rt)/0.99,\ SC_2^5(Rt)/0.98,\ SC_1^6(Rt)/0.97\}.$$

Let us assume that we want to find all semantically consistent subclasses, which simultaneously satisfy configurations D_4 and D_5. This can be done by using Algorithm 1 with the following configuration:

$$D_6 = (Rt/0.94,\ ISD(Rt/0.94),\ N = [5,6],\ M = [0.95,1],\ Q_a = [include = [],\ exclude = []],$$
$$Q_d = [include = [SM_1(Rt/0.94)],\ exclude = [FM_2(Rt.0,94)]]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt/0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_6:

$$D_6(Rt/0.94) = \{SC_1^5(Rt)/0.99,\ SC_1^6(Rt)/0.97\}.$$

And finally, suppose we need to find all semantically consistent subclasses, which simultaneously satisfy configurations D_3 and D_6. This can be done by using Algorithm 1 with the following configuration:

$$D_7 = (Rt/0.94,\ ISD(Rt/0.94),\ N = [5,6],\ M = [0.95,1],$$
$$Q_a = [include = [Rt.p_1/1,\ Rt.f_1/0.9],\ exclude = [Rt.f_2/0.88]],$$
$$Q_d = [include = [SM_1(Rt/0.94)],\ exclude = [FM_2(Rt.0,94)]]).$$

As the result, the algorithm performed the partial decomposition of the class $Rt/0.94$, and constructed the following list of subclasses, which satisfy the decomposition configuration D_7:

$$D_7(Rt/0.94) = \{SC_1^6(Rt)/0.97\}.$$

As we can see, attribute and dependency filtering provide an opportunity to use Algorithm 1 with different configurations for extraction and retrieval of fuzzy conceptual knowledge in a form of fuzzy homogeneous classes of objects. Now let us consider subclass $SC_1^6(Rt)/0.97$ in more detail. According to the definition of the fuzzy homogeneous class of objects $Rt/0.94$, the subclass $SC_1^6(Rt)/0.97$ has the following definition:

$$SC_1^6(Rt)(p_1 = (vertex_1,(v,Pt))/1,$$
$$p_2 = (vertex_2,(v,Pt))/1,$$
$$p_3 = (vertex_3,(v,Pt))/1,$$
$$p_4 = (vertex_4,(v,Pt))/1,$$

$$p_5 = (is\_a\_rectangle, (vf_5(rt), v \in [0,1])) / 0.93,$$
$$f_1 = get\_vertex(rt, (i, \square^+), Pt) / 0.9,$$
$$) / 0.97,$$

where all properties and methods of the subclass have the same meaning as for the class $Rt / 0.94$. As we can see, the subclass $SC_1^6(Rt) / 0.97$ completely satisfies the decomposition configuration D_7 and has a semantically consistent interpretation with the domain of the class $Rt / 0.94$.

Indeed, the subclass $SC_1^6(Rt) / 0.97$ defines four fuzzy points on a plane, which form a fuzzy rectangle, and provides an ability to get access to each vertex of a figure in a form of an object of the fuzzy homogeneous class of objects $Pt / 0.96$. Considered examples of the decomposition of the fuzzy homogeneous class of objects $Rt / 0.94$ demonstrate, that process of the knowledge extraction can be combined with the process of knowledge retrieval, which allows us to consider different strategies for knowledge extraction, retrieval, and integration. For example, if a fuzzy homogeneous class of objects, which is need to be decomposed has a big amount of properties and methods, then it can be fully decomposed only one time, and all constructed semantically consistent subclasses can be integrated into the knowledge base. In this case, the knowledge retrieval process can be reduced to searching within the knowledge base. If a class has a small number of attributes, it can be decomposed each time, when we need to retrieve some of its semantically consistent subclasses. In this case, we can reduce the size of the knowledge base just using the fact, that the class stores all its subclasses within itself.

6. Conclusions

In this paper, we proposed the modified version of the algorithm for the decomposition of fuzzy homogeneous classes of objects, which extracts hidden and non-obvious knowledge in a form of semantically consistent subclasses of a fuzzy homogeneous class of objects based on its internal semantic dependencies. After that, the algorithm performs the knowledge retrieval among subclasses obtained at the previous stage and selects only those ones, which satisfy attribute and dependency filters. Such modification allows the algorithm to reduce the knowledge search space not only in the extraction stage but also in the retrieval stage. In addition, we proposed an approach to the analysis of decomposition consistency, which is based on computing the corresponding coefficients and construction of towers of subclasses lattices. It allows us to consider different strategies for decomposition knowledge extraction and retrieval, which provide new architectural solutions for intelligent knowledge-based systems.

To demonstrate possible application scenarios for a modified version of the decomposition algorithm, we considered the main possible configurations of attribute and dependency filtering parameters. New filtering parameters can be combined with the cardinality and fuzziness of subclasses, which provides a more flexible and powerful tool for describing the retrieval restrictions, as well as reducing knowledge search space. Such an approach to filtering makes the knowledge retrieval process more targetable, especially when we have additional meta-knowledge about the structure and/or behavior of searchable semantically consistent subclasses of the fuzzy homogeneous class of objects. However, despite all advantages of the algorithm, it requires future analysis and optimization.

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